A weak form of self-testing

Jędrzej Kaniewski

Center for Theoretical Physics, Polish Academy of Sciences

(→ Faculty of Physics, University of Warsaw)

www.jkaniewski.eu

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Outline

- Bell nonlocality
- Strong self-testing (CHSH)
- Weak self-testing
- Certifying randomness
- Conclusions and open questions

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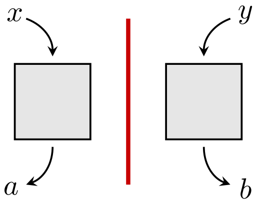
Bell scenario



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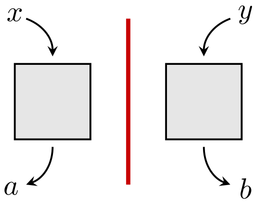

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Assume that $P \in \mathcal{Q}$ is **quantum**

$$P(a, b|x, y) = \langle F_a^x \otimes G_b^y, \rho_{AB} \rangle.$$

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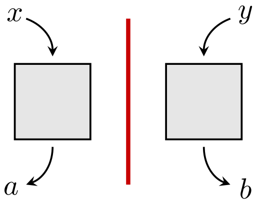

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$$P(a, b|x, y) = \langle F_a^x \otimes G_b^y, \rho_{AB} \rangle.$$

Definition: $P \in \mathcal{L}$ is **local** if

$$P(a, b|x, y) = \sum_{\lambda} p(\lambda)\, p_A(a|x, \lambda)\, p_B(b|y, \lambda).$$

Bell: $\mathcal{L} \subsetneq \mathcal{Q} \iff$ " quantum mechanics is (Bell) **nonlocal** "

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Real vector $C = (c_{abxy})$ define

$$\langle C, P \rangle := \sum_{abxy} c_{abxy} P(a, b|x, y)$$

and

$$\beta_{\mathcal{L}} := \max_{P \in \mathcal{L}} \langle C, P \rangle \quad \text{(local value)}$$

$$\beta_{\mathcal{Q}} := \max_{P \in \mathcal{Q}} \langle C, P \rangle \quad \text{(quantum value)}$$

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Bell violation: $\langle C, P \rangle > \beta_{\mathcal{L}} \implies P \notin \mathcal{L}$

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Can we make this connection more explicit/quantitative?

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- Strong self-testing (CHSH)
- Weak self-testing
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- Conclusions and open questions

Remark 1. Bell functional vs. Bell operator

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$$\langle C, P\rangle = \sum_{abxy} c_{abxy} P(a,b|x,y) = \sum_{abxy} c_{abxy} \langle F_a^x \otimes G_b^y, \rho_{AB}\rangle = \langle W, \rho_{AB}\rangle.$$

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- Proving a bound on the quantum value $\beta_{\mathcal{Q}} \leq c$ is equivalent to showing that

$$W \leq c\,\mathbb{1}$$

for **all possible measurements choices**.

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- This mapping is one-to-one: any A such that

$$A = A^\dagger \quad \text{and} \quad -\mathbb{1} \leq A \leq \mathbb{1}$$

  defines a valid measurement.

CHSH self-testing

- The CHSH operator reads

$$W := A_0 \otimes (B_0 + B_1) + A_1 \otimes (B_0 - B_1),$$

where $-\mathbb{1} \leq A_j \leq \mathbb{1}$ and $-\mathbb{1} \leq B_k \leq \mathbb{1}$.

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- Well known that $\beta_{\mathcal{L}} = 2$ and $\beta_{\mathcal{Q}} = 2\sqrt{2}$.

"The maximal violation $\beta = 2\sqrt{2}$ can be achieved in an essentially unique manner"

[Tsirelson '87], [Summers and Werner '87], [Popescu and Rohrlich '92]

CHSH self-testing

Proof:

- Define

$$V_0 = A_0 \otimes \mathbb{1} - \mathbb{1} \otimes \frac{B_0 + B_1}{\sqrt{2}},$$

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- Check

$$W = \frac{1}{\sqrt{2}}\Big[(A_0^2 + A_1^2) \otimes \mathbb{1} + \mathbb{1} \otimes (B_0^2 + B_1^2) - (V_0^\dagger V_0 + V_1^\dagger V_1)\Big].$$

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- Conclude that $W \leq 2\sqrt{2}\,\mathbb{1} \implies \mathrm{tr}(W\rho_{AB}) \leq 2\sqrt{2} = \beta_{\mathcal{Q}}$, so the SOS decomposition is **tight**.

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If ρ_A and ρ_B are full-rank, then

$$A_0^2 = \mathbb{1} \implies \left(\frac{B_0 + B_1}{\sqrt{2}}\right)^2 = \mathbb{1} \implies \{B_0, B_1\} = 0.$$

CHSH self-testing

- These algebraic relations determine the form of observables

$$B_0^2 = B_1^2 = \mathbb{1} \quad \text{and} \quad \{B_0, B_1\} = 0 \implies \begin{array}{c} B_0 = U_B(\sigma_x \otimes \mathbb{1})U_B^\dagger \\ B_1 = U_B(\sigma_z \otimes \mathbb{1})U_B^\dagger \end{array}$$

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- Construct W and determine the eigenspace corresponding to $\lambda = 2\sqrt{2}$ (essentially a two-qubit operator).

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Self-testing (rigidity) statement for CHSH: if $\beta = 2\sqrt{2}$ then

$$\begin{array}{ll} A_0 = U_A(\sigma_x \otimes \mathbb{1})U_A^\dagger & \qquad B_0 = U_B(\sigma_x \otimes \mathbb{1})U_B^\dagger \\ A_1 = U_A(\sigma_z \otimes \mathbb{1})U_A^\dagger & \qquad B_1 = U_B(\sigma_z \otimes \mathbb{1})U_B^\dagger \end{array}$$

and

$$\rho_{AB} = U(\Phi_{A'B'} \otimes \tau_{A''B''})U^\dagger \quad \text{for} \quad U := U_A \otimes U_B$$

CHSH self-testing

Strategy:

1. Find tight SOS decomposition.
2. Deduce algebraic relations between local observables.
3. Deduce their exact form (up to unitaries and extra degrees of freedom).
4. Construct Bell operator and find eigenspace corresponding to $\beta_{\mathcal{Q}}$.

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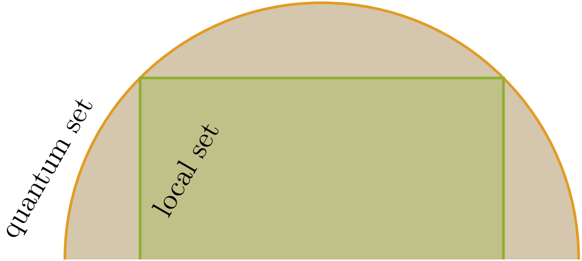
Immediate consequence of self-testing: the maximal violation is achieved by a unique probability point



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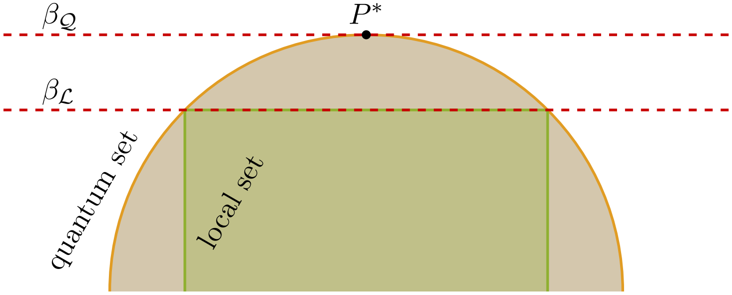

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- In a scenario with 3 inputs and 2 outputs per party consider

$$W = A_0 \otimes (B_0 + B_1 + B_2) + A_1 \otimes (B_0 + B_1 - B_2) + A_2 \otimes (B_0 - B_1).$$

(the correlation part of the infamous I_{3322} functional)

- Easy to show that $\beta_{\mathcal{L}} = 4$ and $\beta_{\mathcal{Q}} = 5$, but the maximal violation is achieved by multiple probability points.

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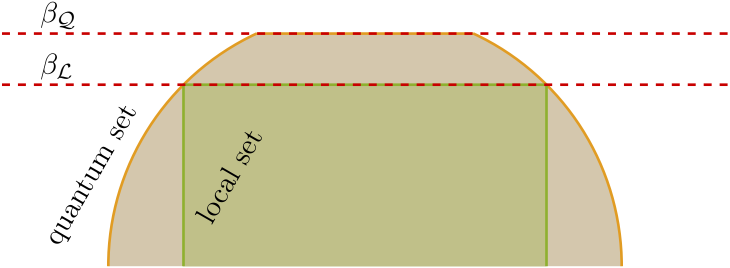

no unique maximiser $\implies$ no rigidity statement

can we still have some **weak form of self-testing**?

A weak form of self-testing

- We already have a tight SOS decomposition

$$2W = (2A_0^2 + 2A_1^2 + A_2^2) \otimes \mathbb{1} + \mathbb{1} \otimes (2B_0^2 + 2B_1^2 + B_2^2) - \sum_{j=0}^{2} V_j^\dagger V_j,$$

where

$$\begin{aligned} V_0 &= (A_0 + A_1) \otimes \mathbb{1} - \mathbb{1} \otimes (B_0 + B_1), \\ V_1 &= (A_0 - A_1) \otimes \mathbb{1} - \mathbb{1} \otimes B_2, \\ V_2 &= A_2 \otimes \mathbb{1} - \mathbb{1} \otimes (B_0 - B_1). \end{aligned}$$

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- Observing $\beta = 5$ implies

$$\operatorname{tr}(A_j^2 \rho_A) = \operatorname{tr}(B_j^2 \rho_B) = 1,$$
$$V_j \rho_{AB} = 0$$

for $j = 0, 1, 2$.

A weak form of self-testing

- Let us derive an explicit form of the observables. Rewriting $V_1 \rho_{AB} = 0$ gives

$$\big[(A_0 - A_1) \otimes \mathbb{1}\big]\rho_{AB} = (\mathbb{1} \otimes B_2)\rho_{AB},$$

which combined with the full-rank assumption leads to

$$(A_0 - A_1)^2 = \mathbb{1}.$$

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- Together with projectivity this gives

$$\begin{aligned}\mathcal{H}_A &\equiv \mathbb{C}^2 \otimes \mathbb{C}^{d_A},\\ A_0 &= \Big(\cos\frac{\pi}{6}\,\mathsf{X} + \sin\frac{\pi}{6}\,\mathsf{Z}\Big) \otimes \mathbb{1},\\ A_1 &= \Big(\cos\frac{\pi}{6}\,\mathsf{X} - \sin\frac{\pi}{6}\,\mathsf{Z}\Big) \otimes \mathbb{1}.\end{aligned}$$

(up to a choice of basis)

A weak form of self-testing

- Combining $V_0\rho_{AB} = 0$ and $V_2\rho_{AB} = 0$ gives
$$\frac{1}{2}\big[(A_0 + A_1 + A_2) \otimes \mathbb{1}\big]\rho_{AB} = (\mathbb{1} \otimes B_2)\rho_{AB}$$
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- Simple algebra yields
$$A_2 = \sum_{j=1}^{d_A} \big(\cos u_j\,\mathsf{Y} + \sin u_j\,\mathsf{Z}\big) \otimes |e_j\rangle\langle e_j|,$$
where $u_j \in [0, 2\pi)$ and $\{|e_j\rangle\}_{j=1}^{d_A}$ is an orthonormal basis on $\mathbb{C}^{d_A}$.

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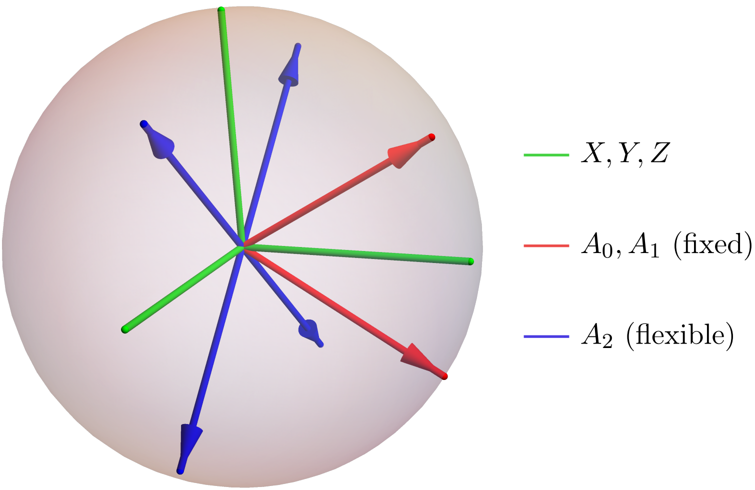

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- By symmetry the same characterisation holds for the observables of Bob. The Bell operator reads

$$W = \sum_{j=1}^{d_A} \sum_{k=1}^{d_B} R(u_j, v_k) \otimes |e_j\rangle\langle e_j| \otimes |f_k\rangle\langle f_k|,$$

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- The corresponding eigenvector is the standard maximally entangled state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ (**smart** parameterisation)
- Finally, the global state must be of the form

$$\rho_{AB} = \Phi^+_{A'B'} \otimes \sigma_{A''B''},$$

  where $\sigma_{A''B''}$ is a normalised state satisfying

$$\mathrm{tr}\left(\sigma_{A''B''} |e_j\rangle\langle e_j| \otimes |f_k\rangle\langle f_k|\right) = 0 \quad \text{if} \quad u_j \neq v_k.$$

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Conclusion: there is a family of optimal strategies parametrised by $u \in [0, 2\pi)$ and **every optimal strategy is just a convex combination** of those.

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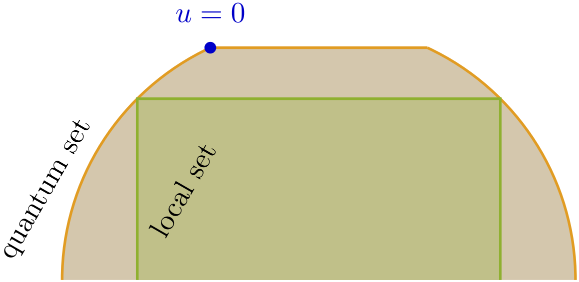

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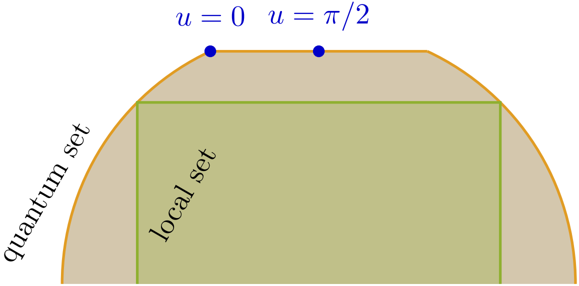

A weak form of self-testing

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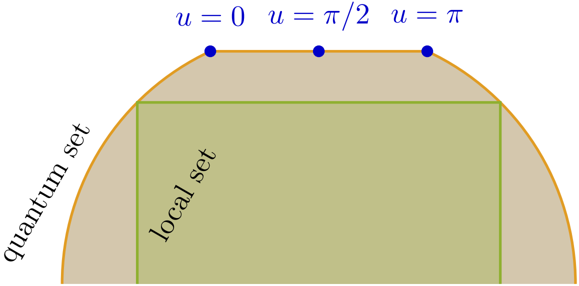

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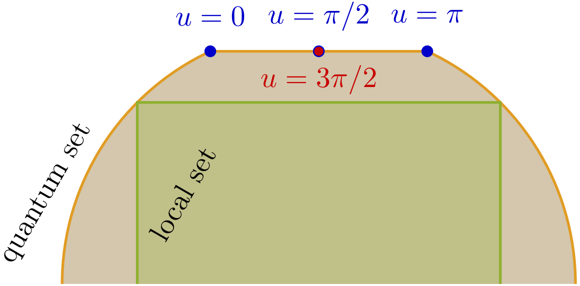

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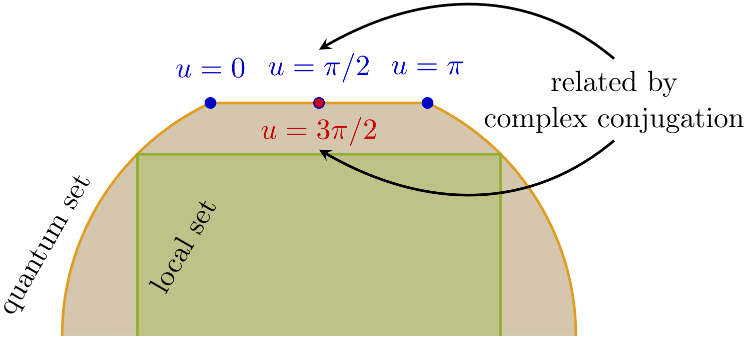

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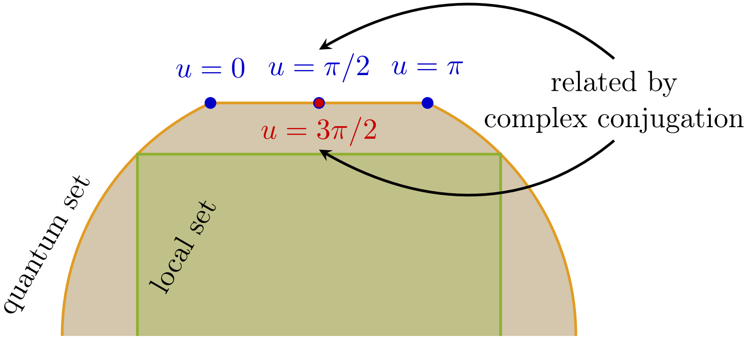

1. The face is a line segment.
2. The endpoints are self-tests in the usual strong sense.

Outline

- Bell nonlocality
- Strong self-testing (CHSH)
- Weak self-testing
- Certifying randomness
- Conclusions and open questions

Certifying randomness

Setup: Alice and Bob observe Bell violation, Eve is trying to guess the outcome of Alice for a specific setting. We start with A_2:

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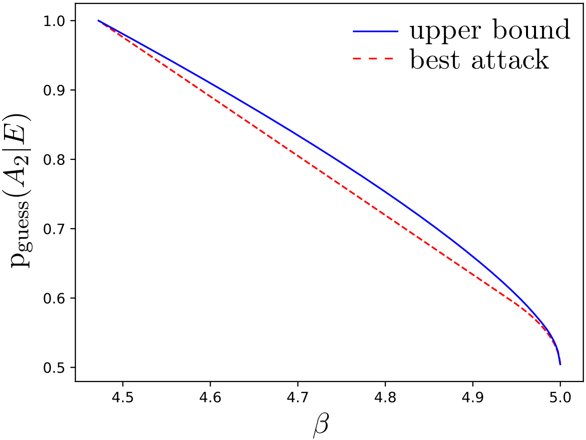

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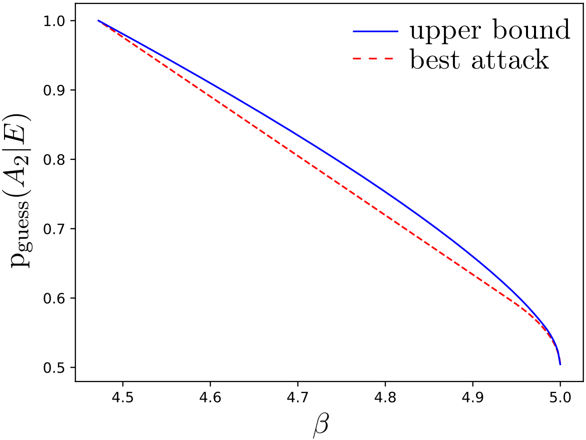

randomness guaranteed only when $\beta > 4.5$

$(\beta_{\mathcal{L}} = 4)$

Certifying randomness

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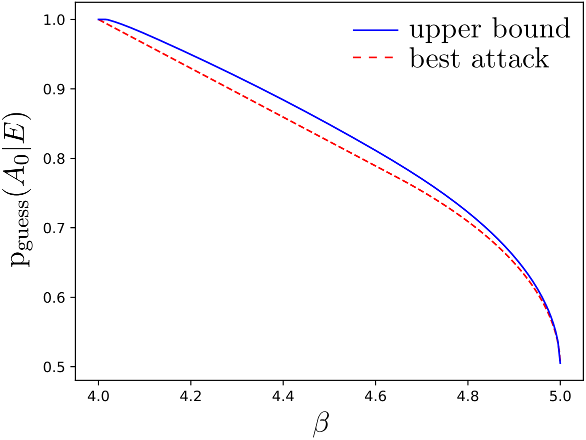

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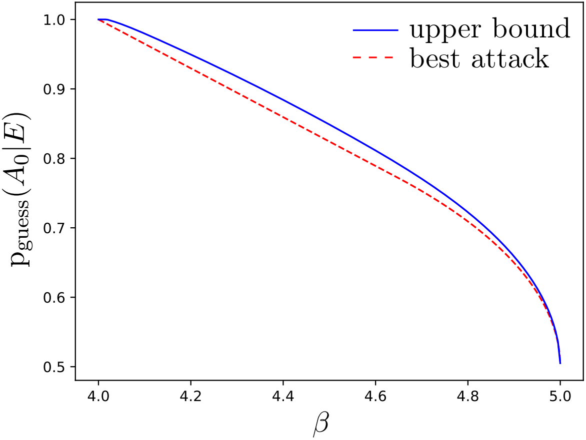

randomness guaranteed for all $\beta > 4$

randomness profile similar to CHSH

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Thank you for your attention!