Secure bit commitment from relativistic constraints

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joint work with Marco Tomamichel, Esther Hänggi and Stephanie Wehner

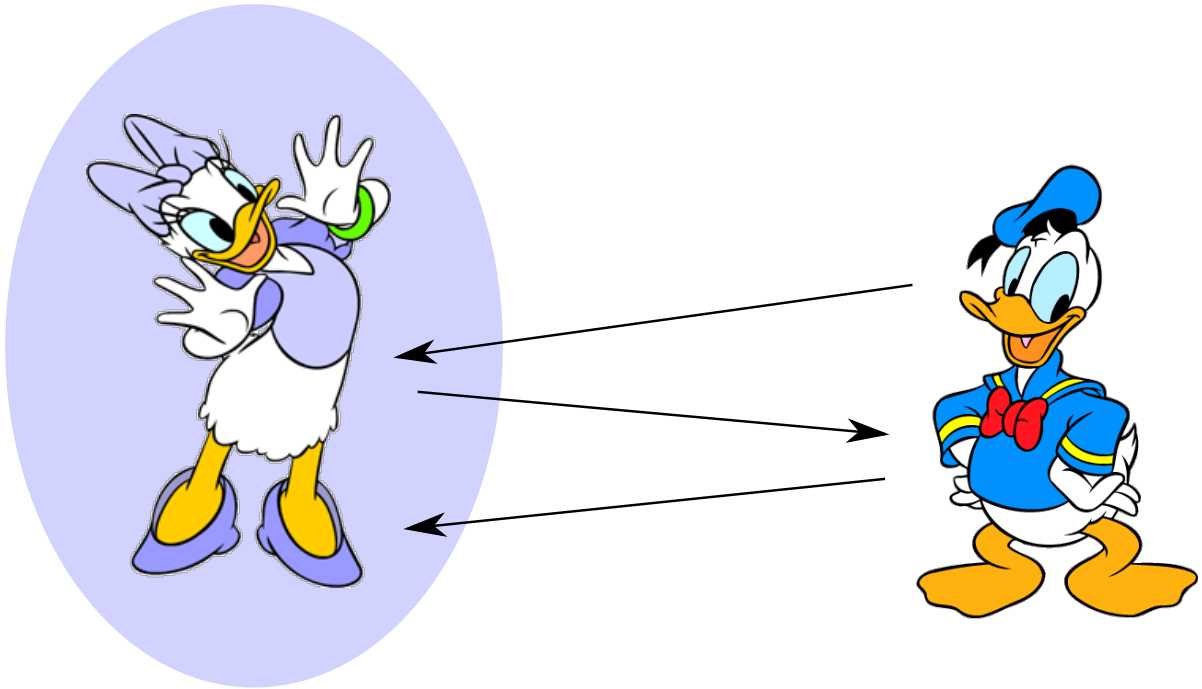
[arXiv: 1206.1740]

QCrypt 2012, Singapore

Outline

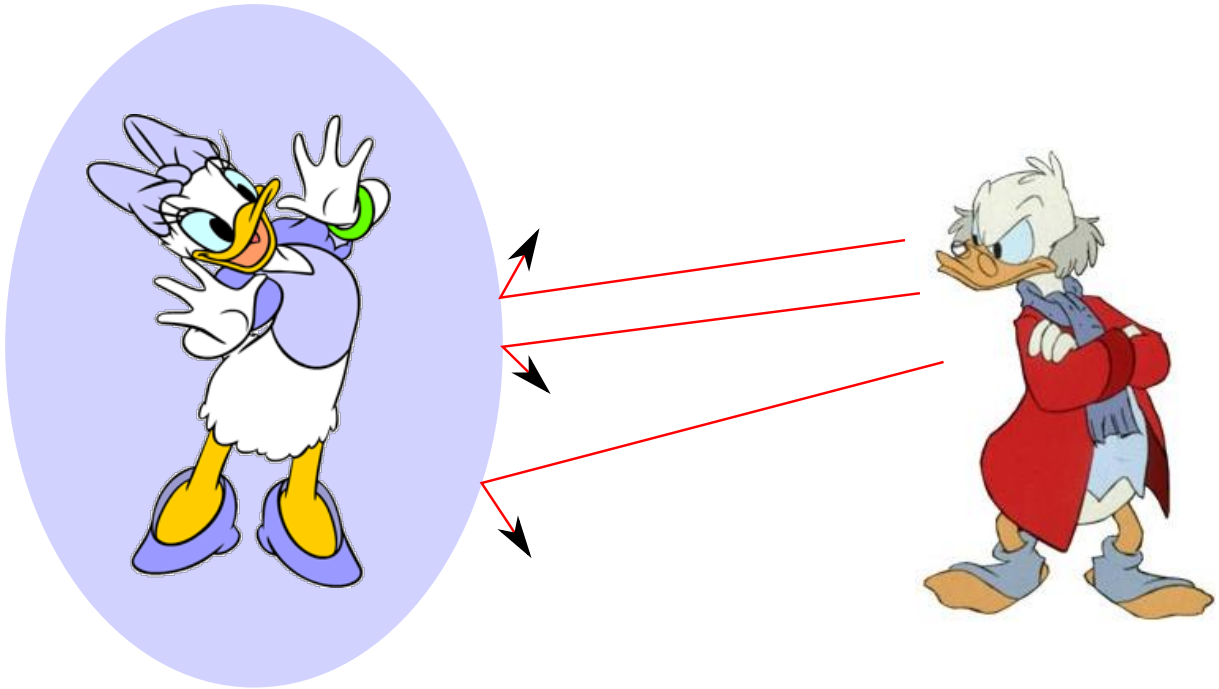
- Two-party cryptographic primitives
- Classical and quantum bit commitment
- Relativistic setting
- Relativistic bit commitment protocol [Kent'11]
- Security proof
- Summary and open questions

Two-party crypto – concept



both honest $\Longrightarrow$ protocol goes through and result is as expected

Two-party crypto – concept



Alice is honest $\Longrightarrow$ she is protected against dishonest Bob (e.g. she catches him cheating, aborts the protocol or he remains ignorant about her input) and *vice versa*

Two-party crypto – examples

secure function evaluation

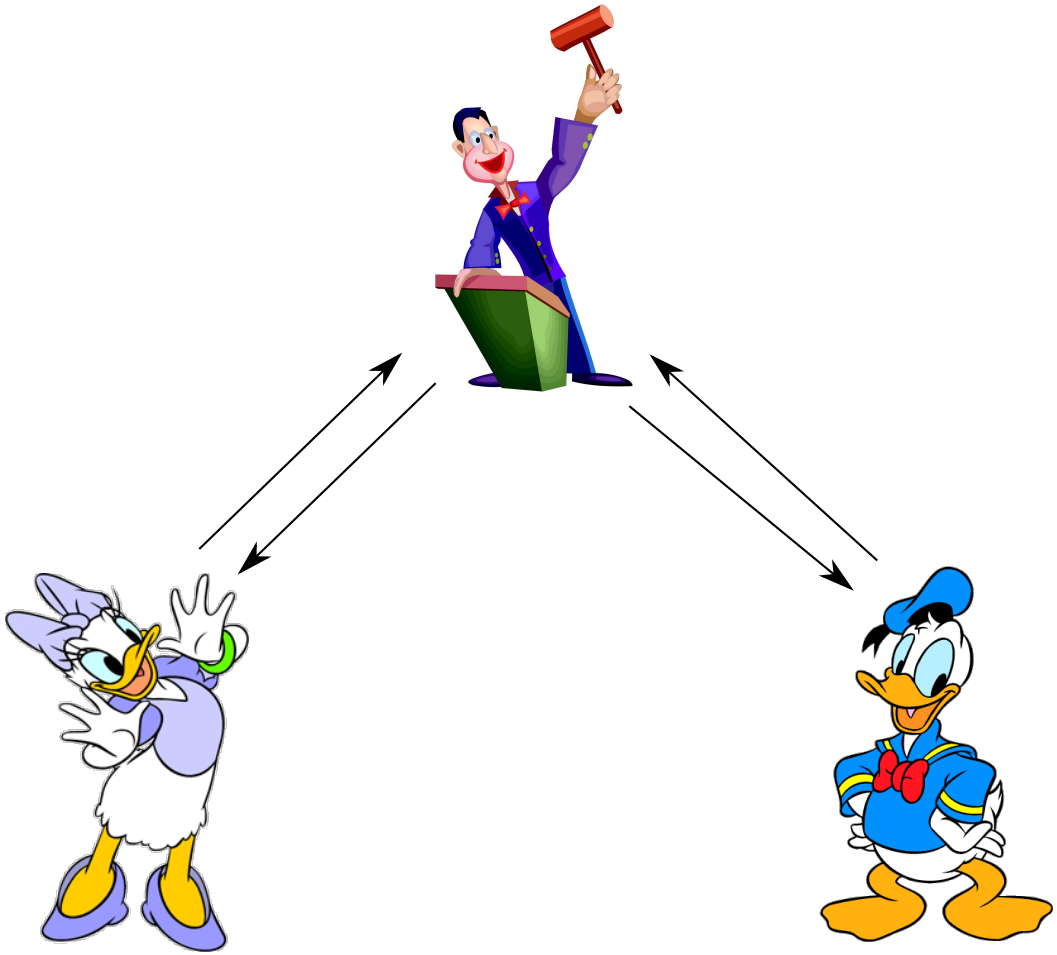
oblivious transfer

coin flip

(trusted, unbiased randomness)

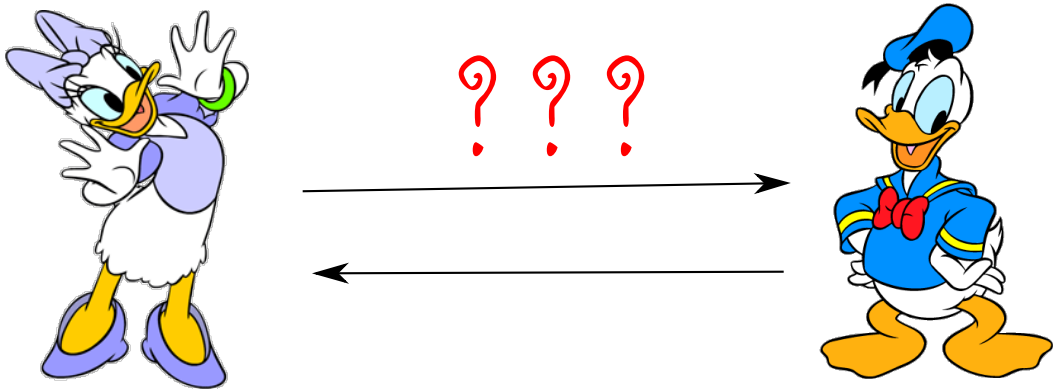
commitment schemes

Auction – motivation for commitment schemes



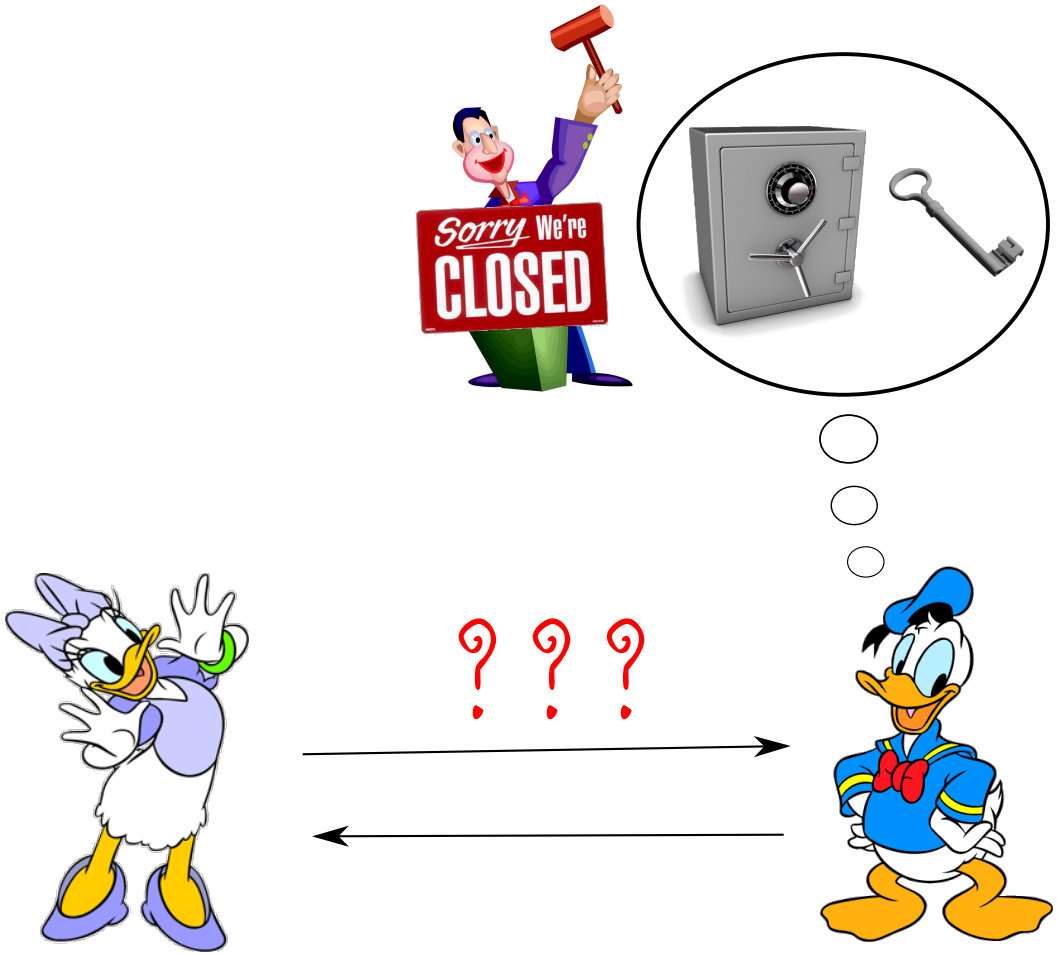
Auctioning is easy if a trusted third-party is available.

Auction – motivation for commitment schemes



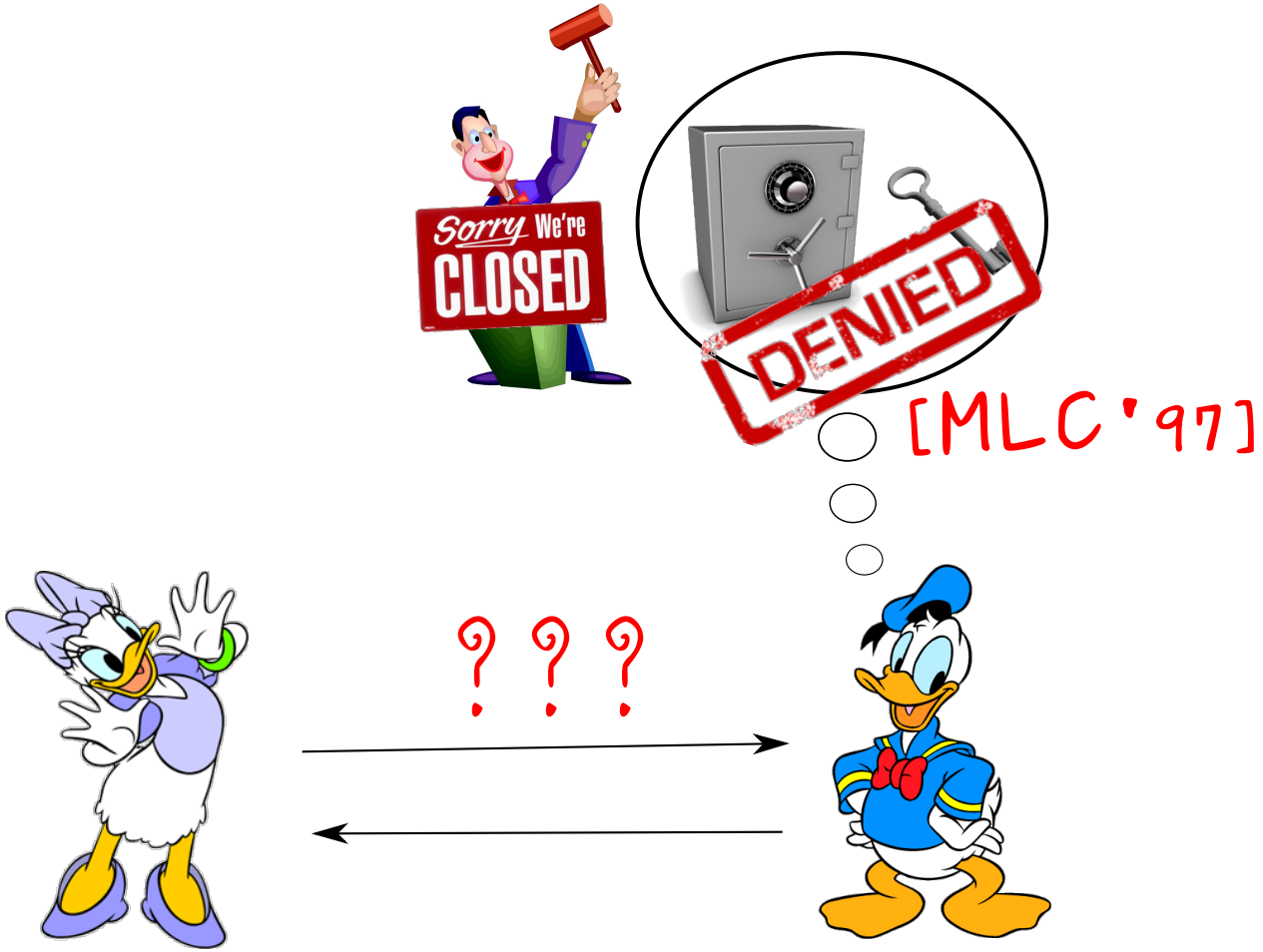
What if there is no trusted third-party? Be paranoid, trust nobody!

Auction – motivation for commitment schemes



We could do it if we had a perfect [information-theoretic] safe.

Auction – motivation for commitment schemes



Is this it?

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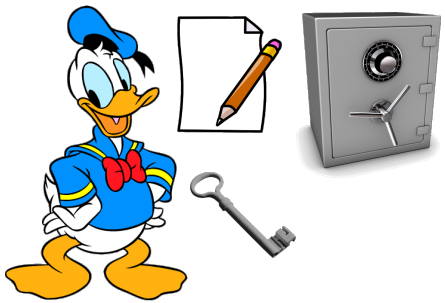
- Relativistic bit commitment protocol [Kent'11]
- Security proof
- Summary and open questions

Bit commitment – ideal functionality

Commit phase



Bit commitment – ideal functionality

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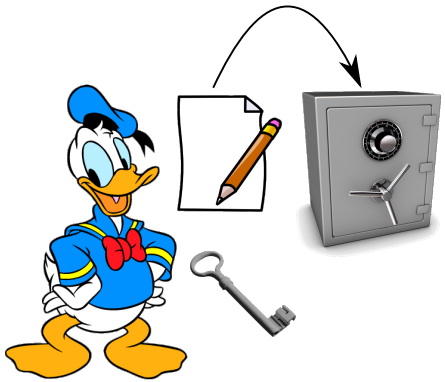

Bit commitment – ideal functionality

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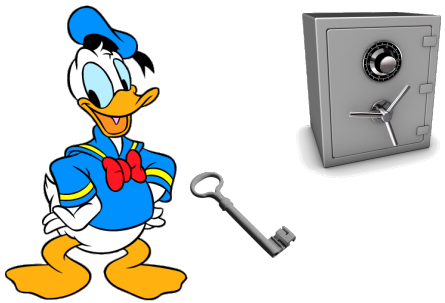

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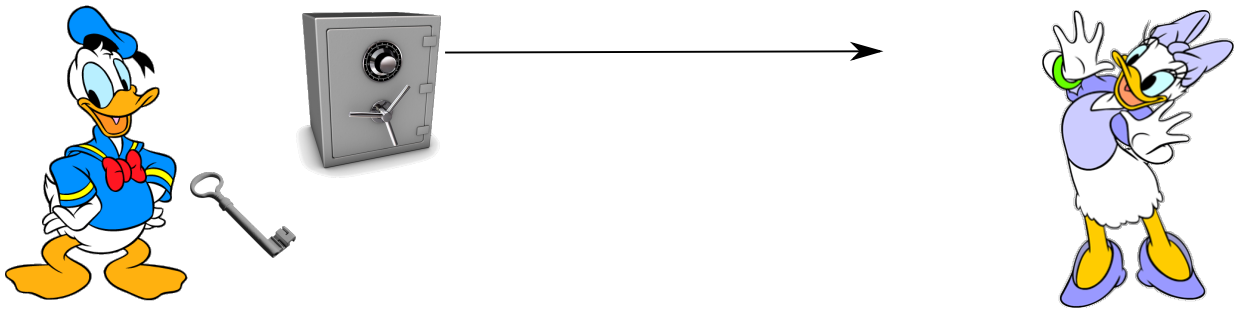

Bit commitment – ideal functionality

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Bit commitment – ideal functionality

Commit phase

Open phase

Bit commitment – ideal functionality

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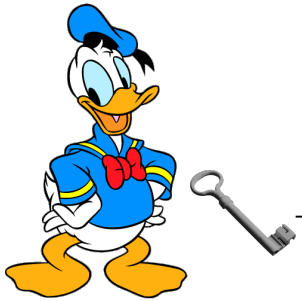


Bit commitment – ideal functionality

Commit phase

Open phase

Bit commitment – ideal functionality

Commit phase

Open phase

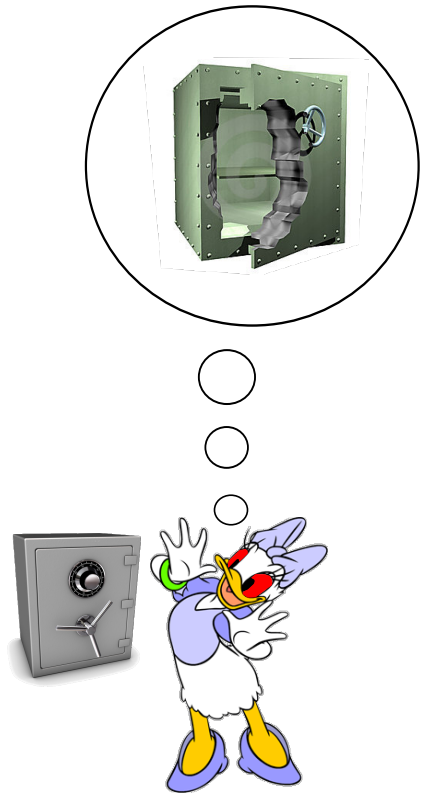
Cheating objectives

The commit phase is over...

Cheating objectives

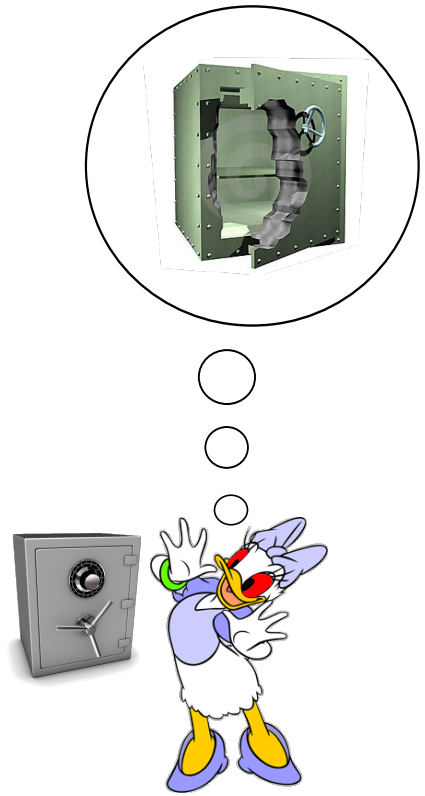
Alice goes mad!

Cheating objectives



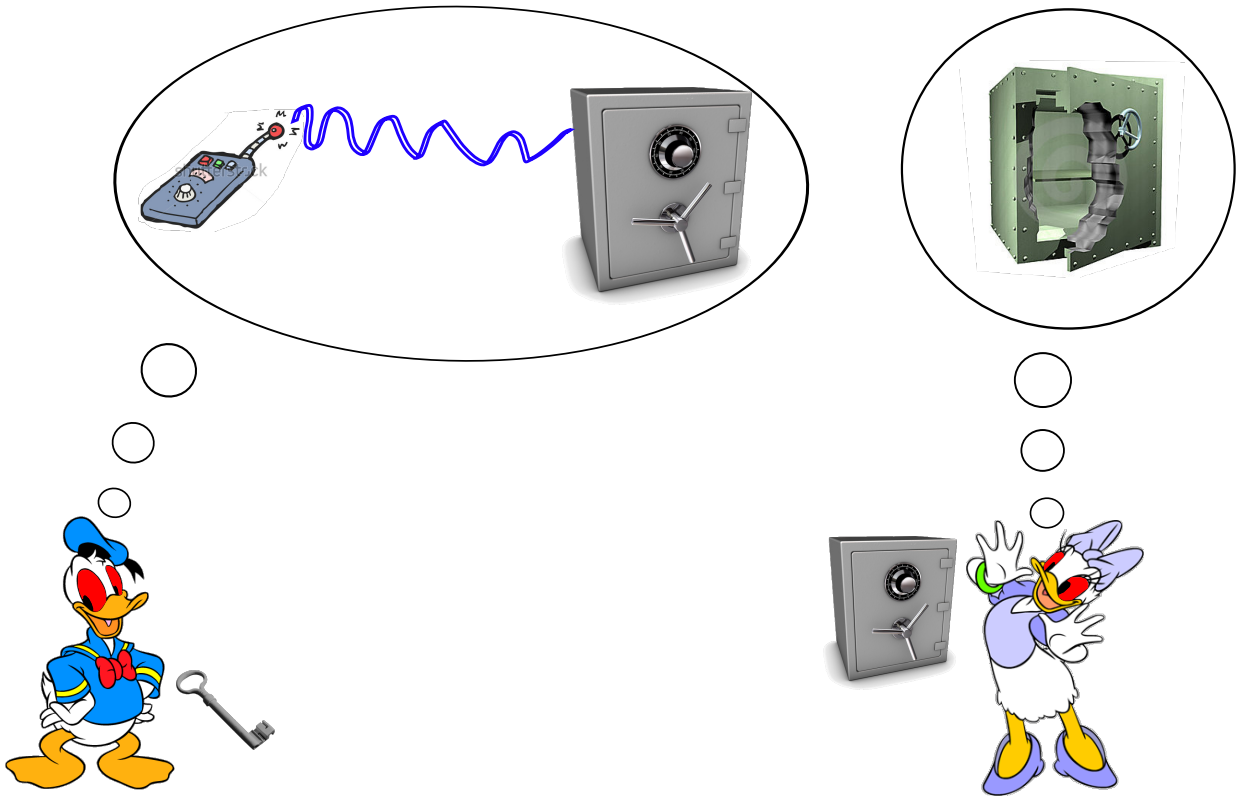
She wants to break the safe and read the message!

Cheating objectives



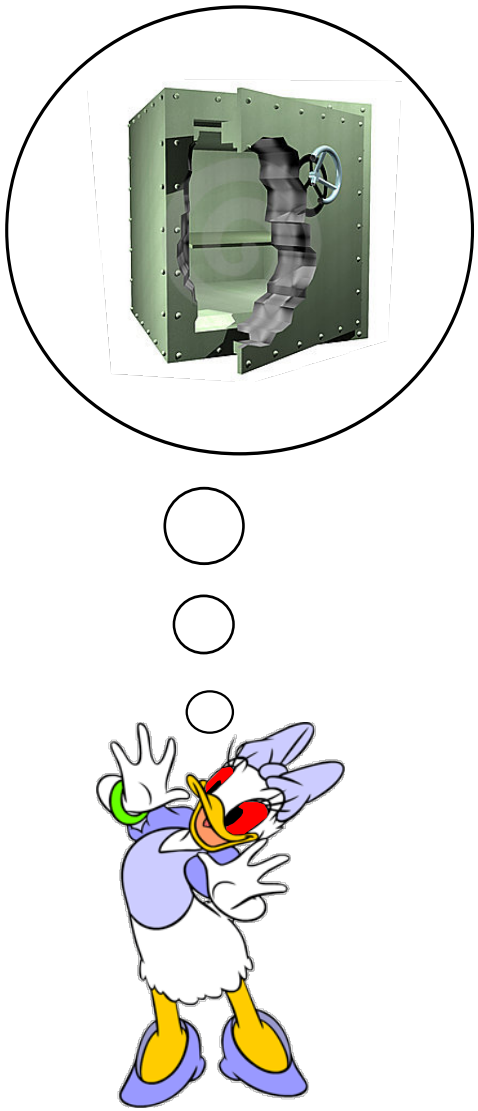
Bob goes mad!

Cheating objectives



He wants to influence the message, he wants to be uncommitted!

Security criteria



The scheme should be **hiding**.

p_{guess} – probability that Alice guesses the commited bit correctly after the commit phase is over

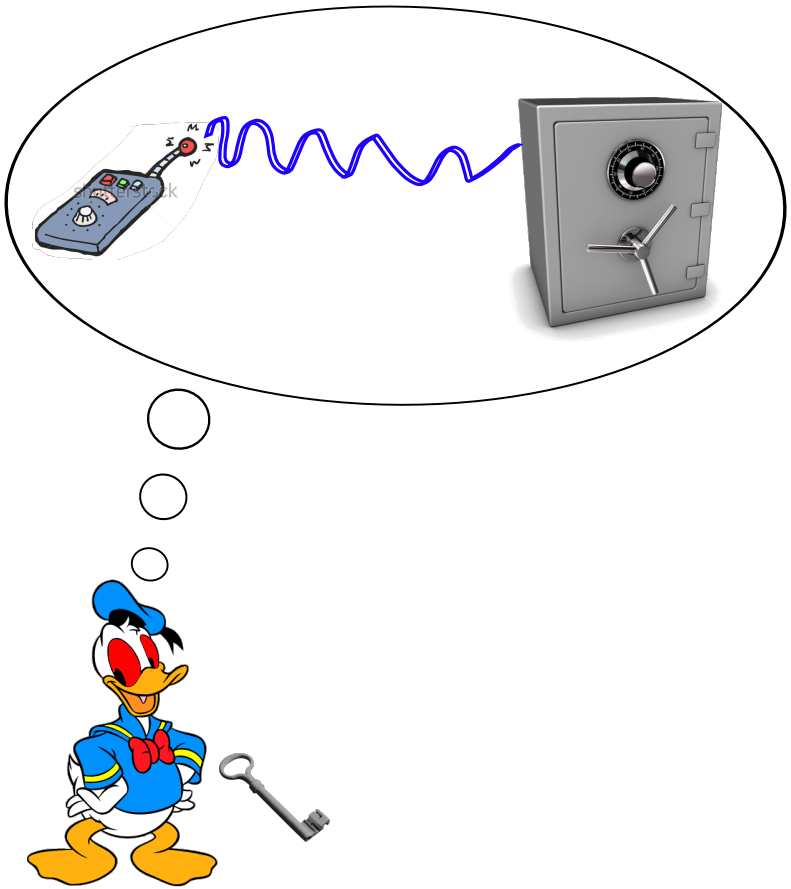
Definition

*A bit commitment protocol is δ-**hiding** if the fact that Bob is honest implies*

$$p_{\text{guess}} \leq \frac{1}{2} + \delta.$$

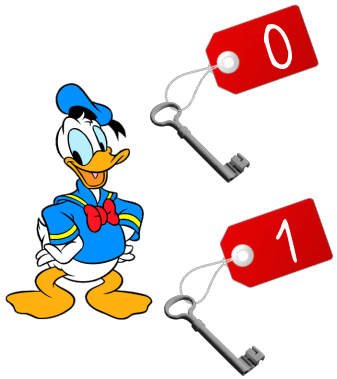
Security criteria

The scheme should be **binding**.



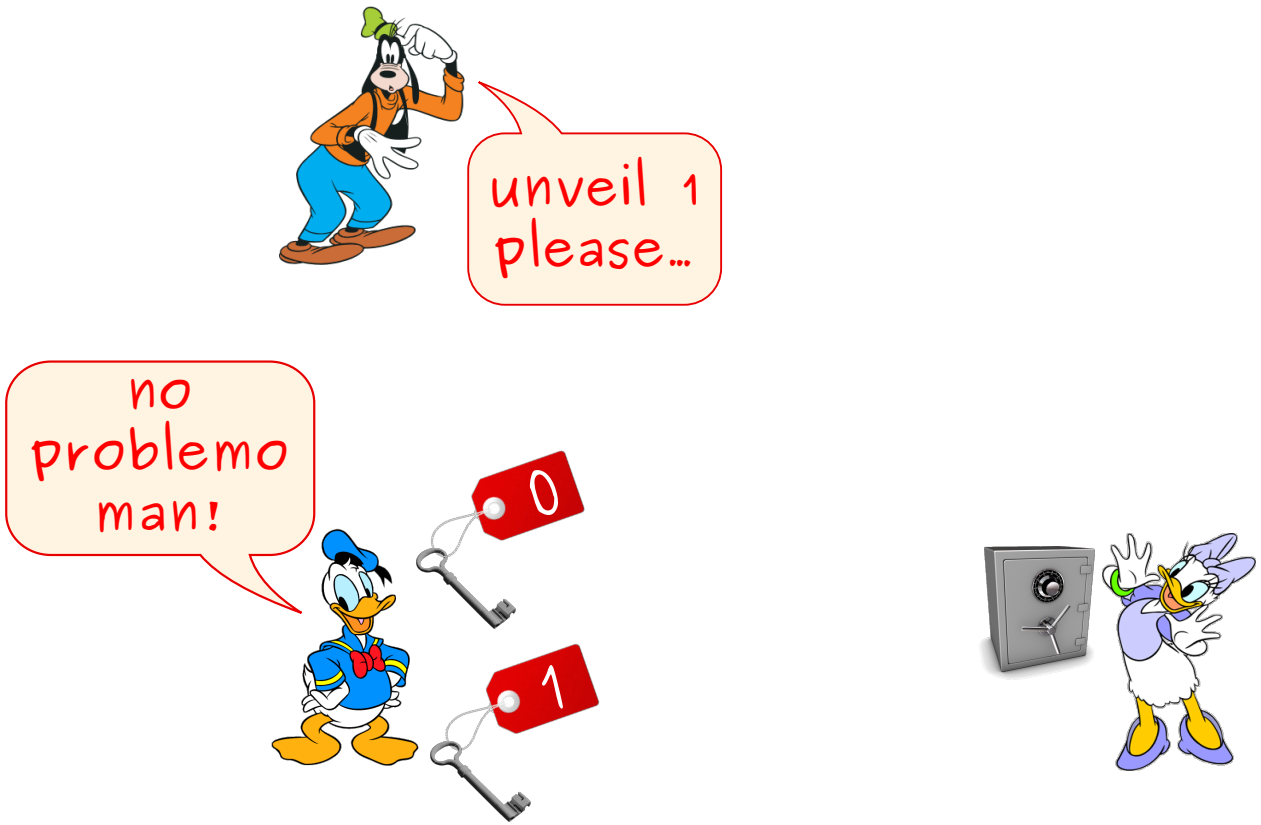
Bob should not be able to change his mind after the commit phase is over.

Security criteria



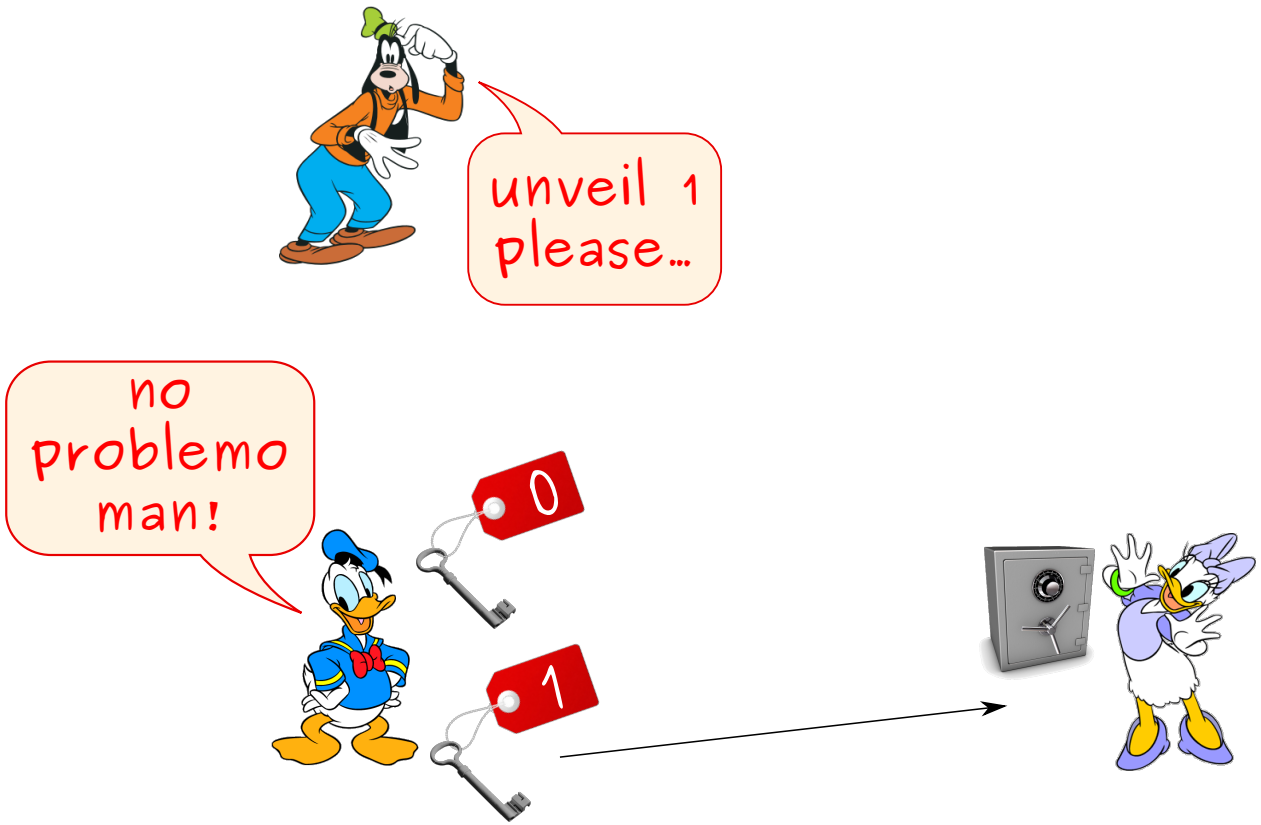
Dishonest Bob will have two different keys...

Security criteria



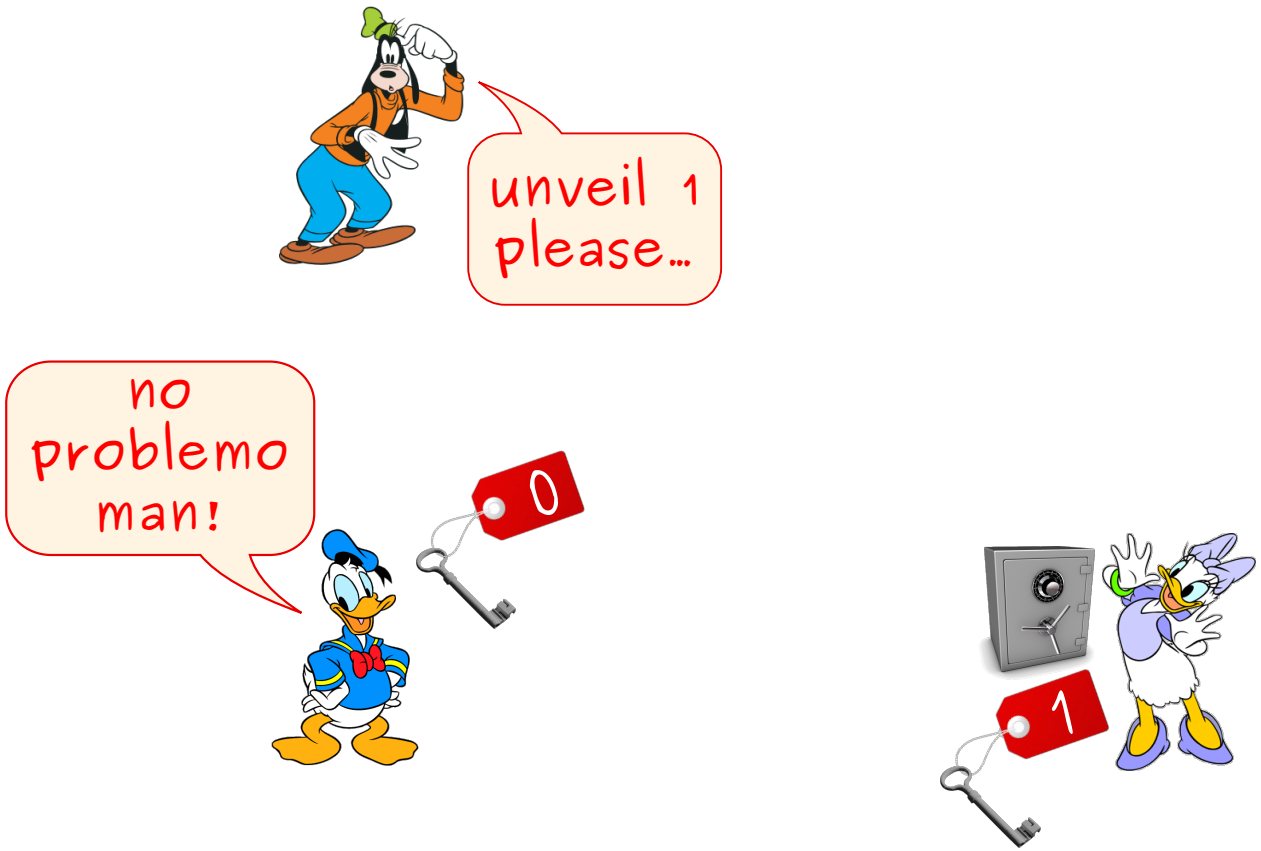
External verifier Victor asks him to unveil 1.

Security criteria



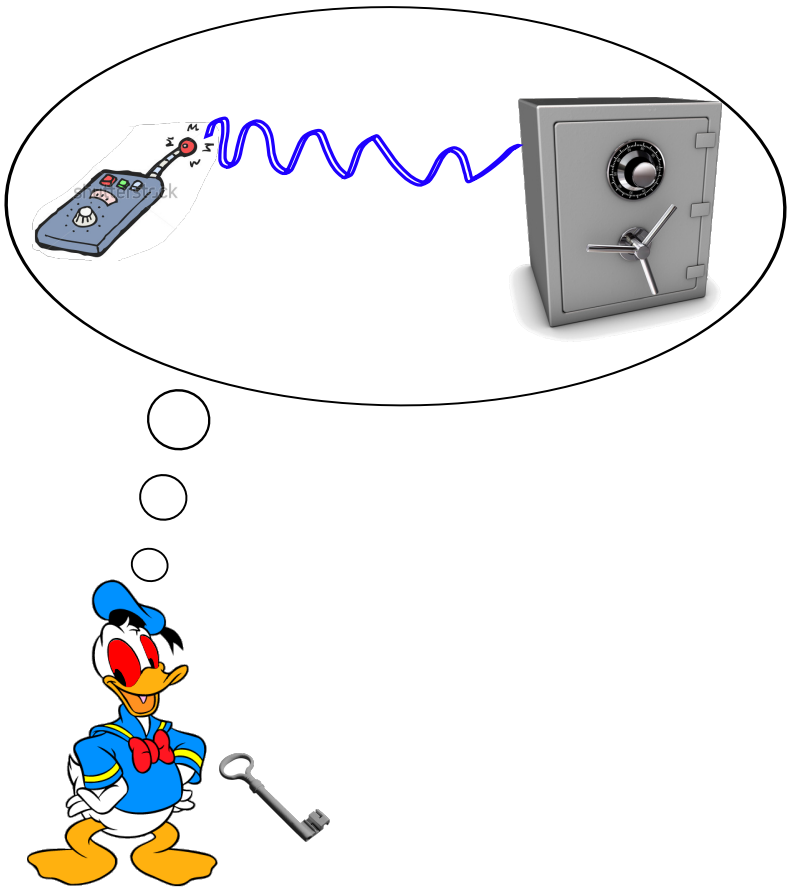
Bob attempts to unveil 1.

Security criteria



p_1 is the probability that Alice accepts the unveiling.

Security criteria



Definition

A bit commitment protocol is **ϵ-binding** *if the fact that Alice is honest implies that there exists a bit $c \in \{0, 1\}$ such that $p_c \leq \epsilon$.*

What about **superposition** commitment?

For any protocol Bob can commit to an honest superposition and achieve $p_0 = p_1 = \frac{1}{2}$.

Not satisfiable in the quantum world...

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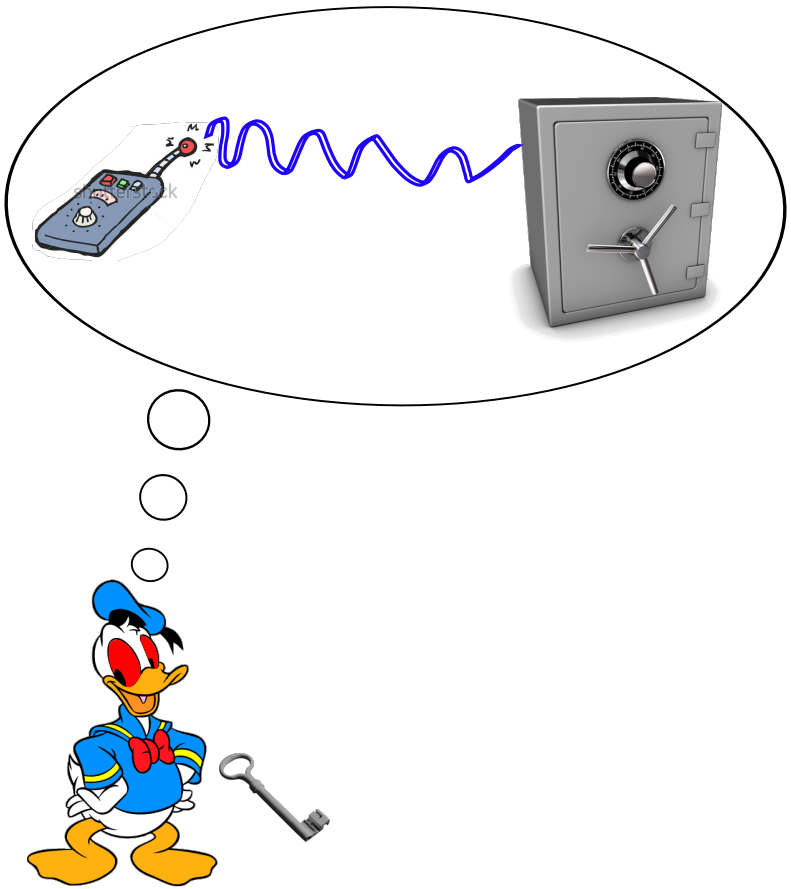

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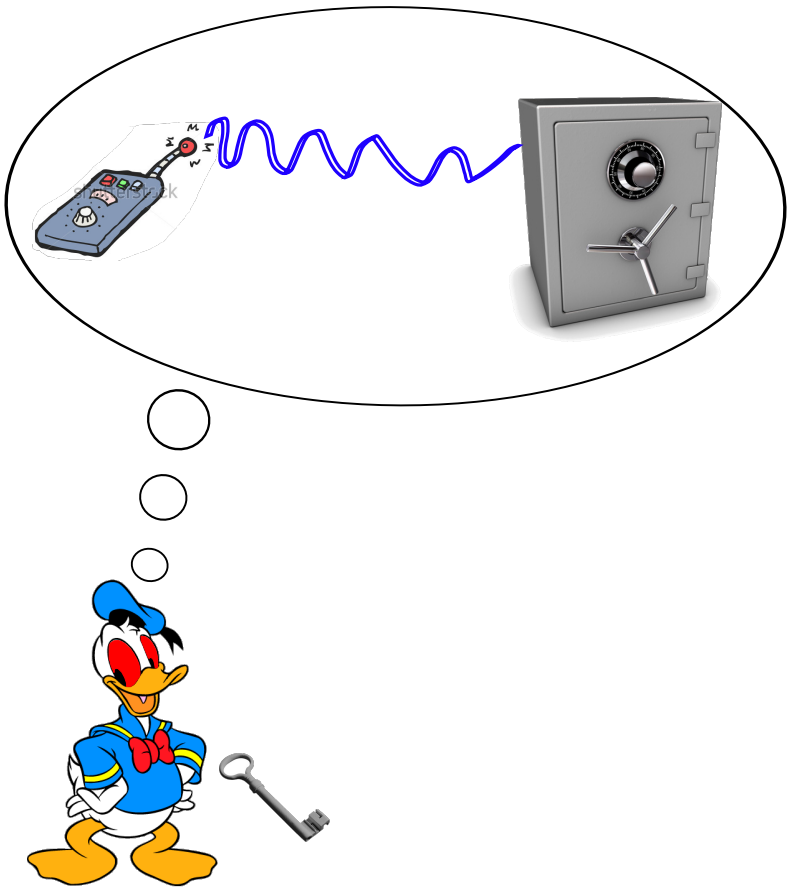

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A bit commitment protocol is ϵ**-weakly binding** *if the fact that Alice is honest implies that* $p_0 + p_1 \leq 1 + \epsilon$.

Composability? forget it...

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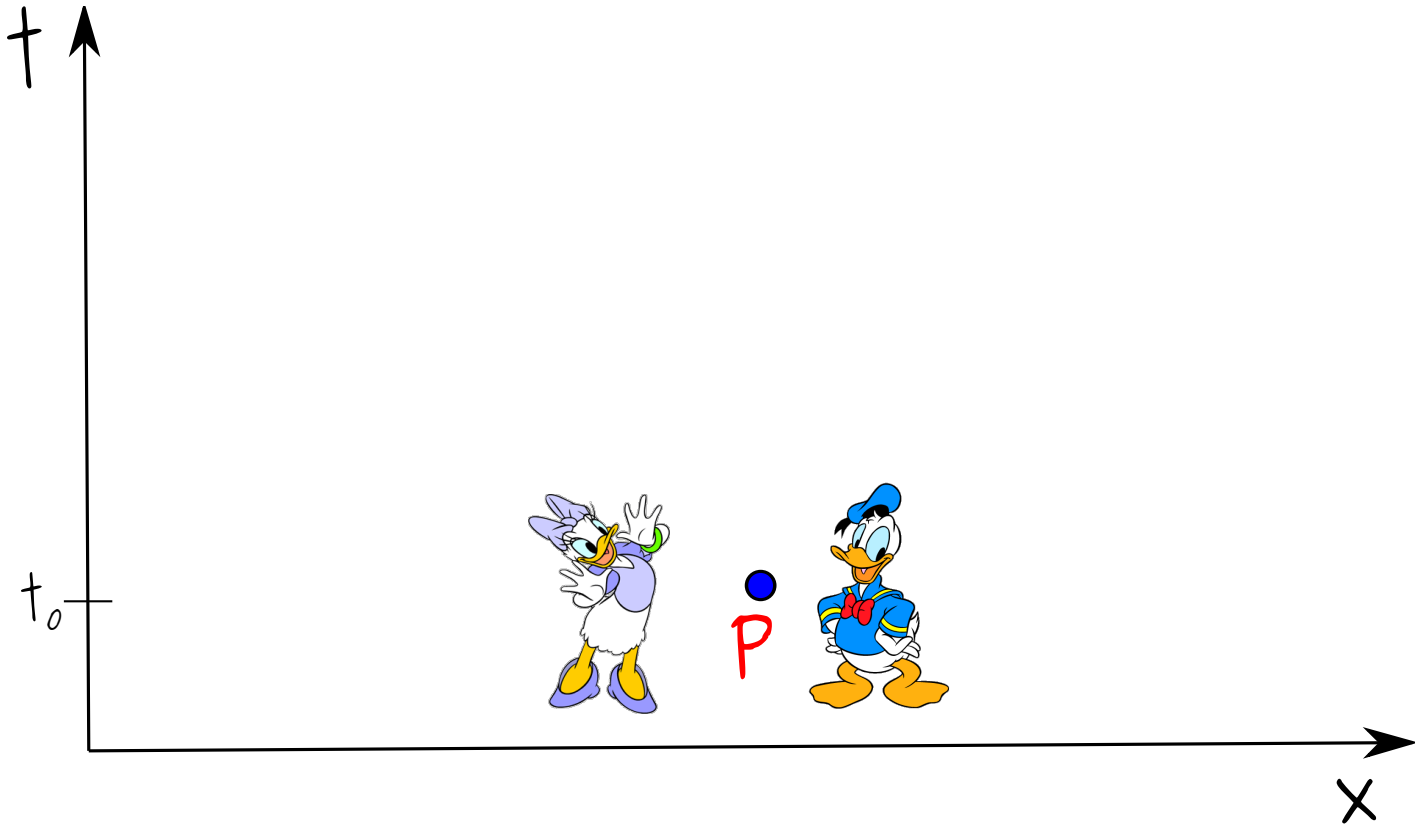
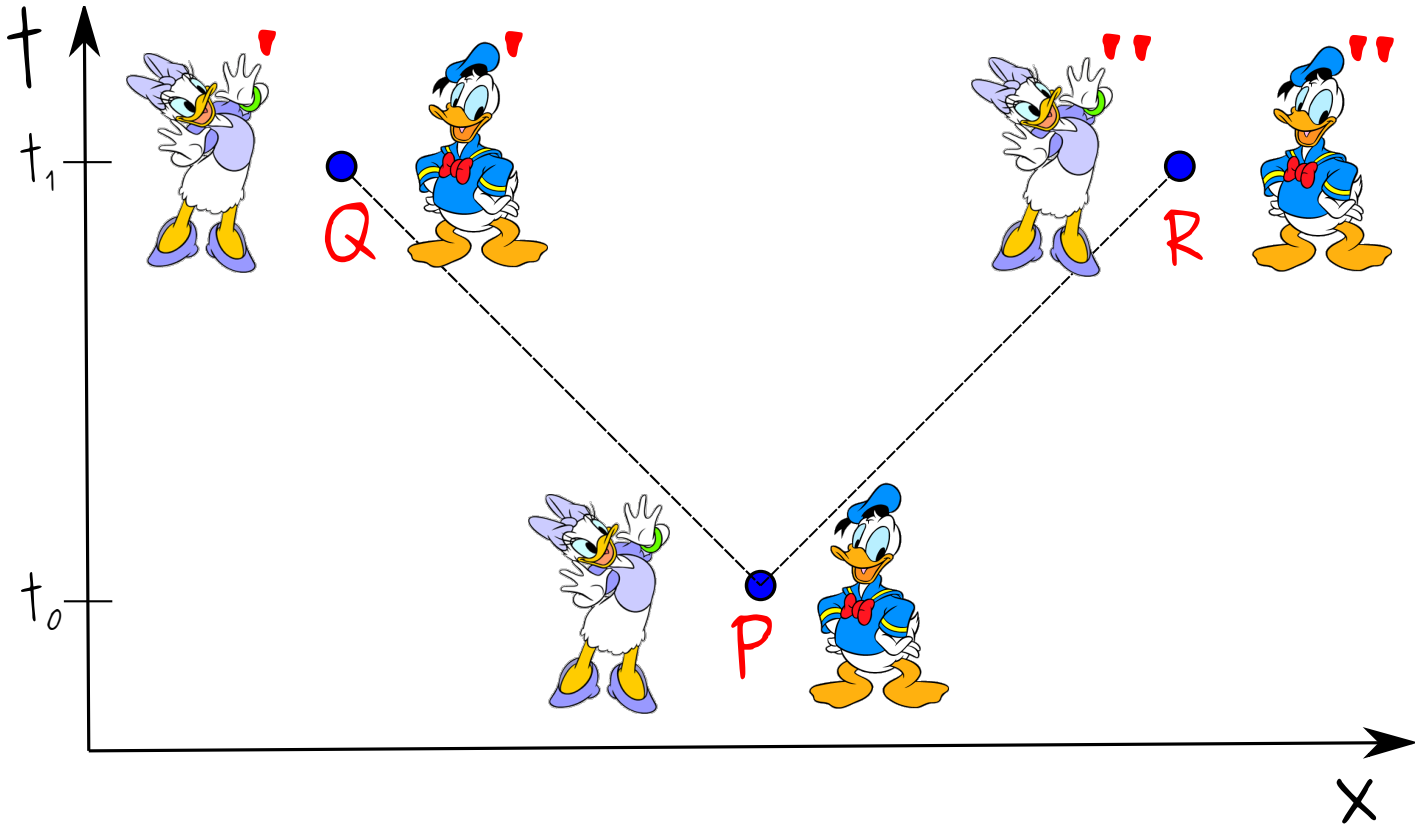
Relativistic setting

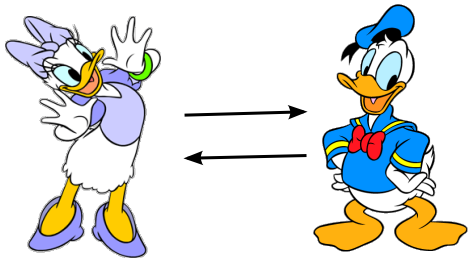


Relativistic setting



Relativistic setting

Phase 1



Relativistic setting

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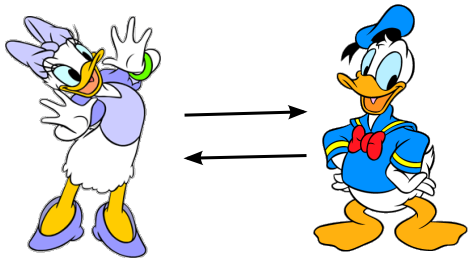
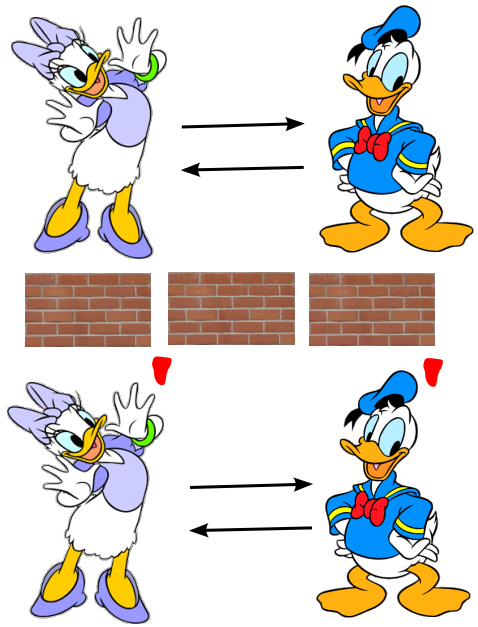


Phase 2



Relativistic setting

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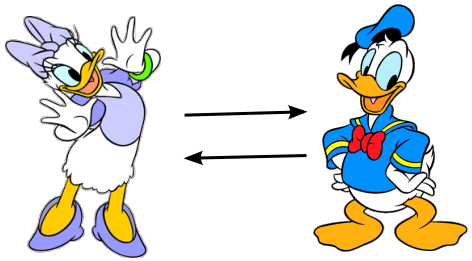

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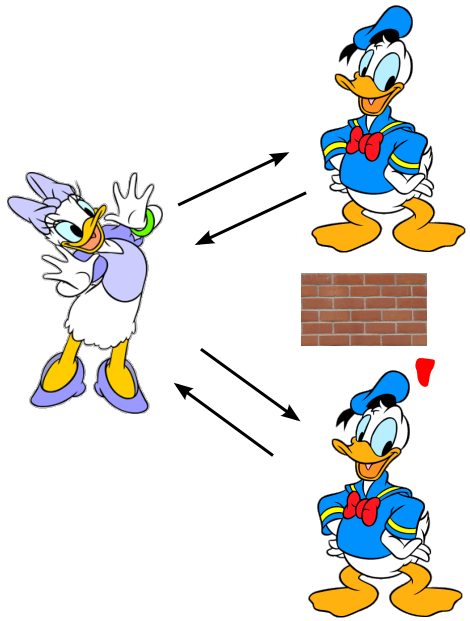
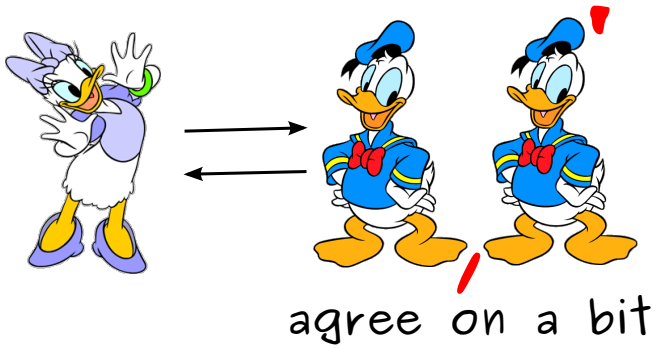
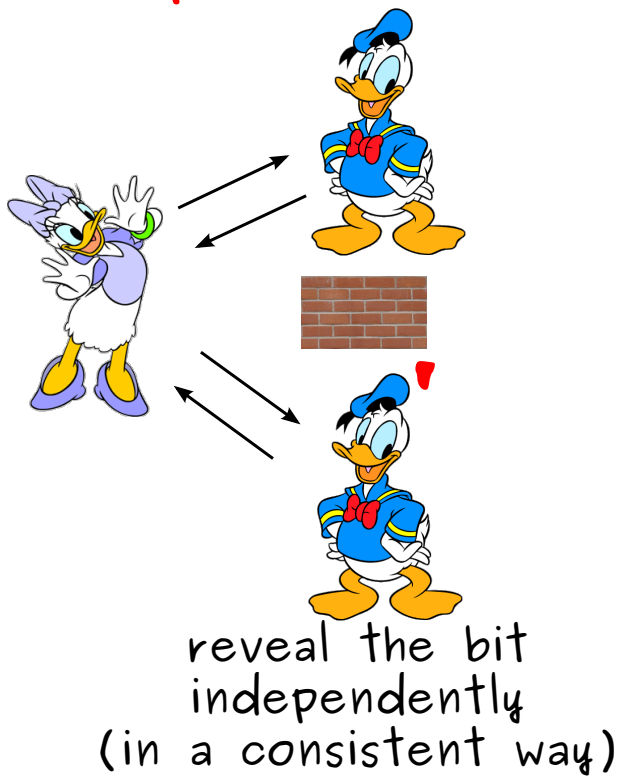


Relativistic setting

Phase 1
commit



Phase 2
open



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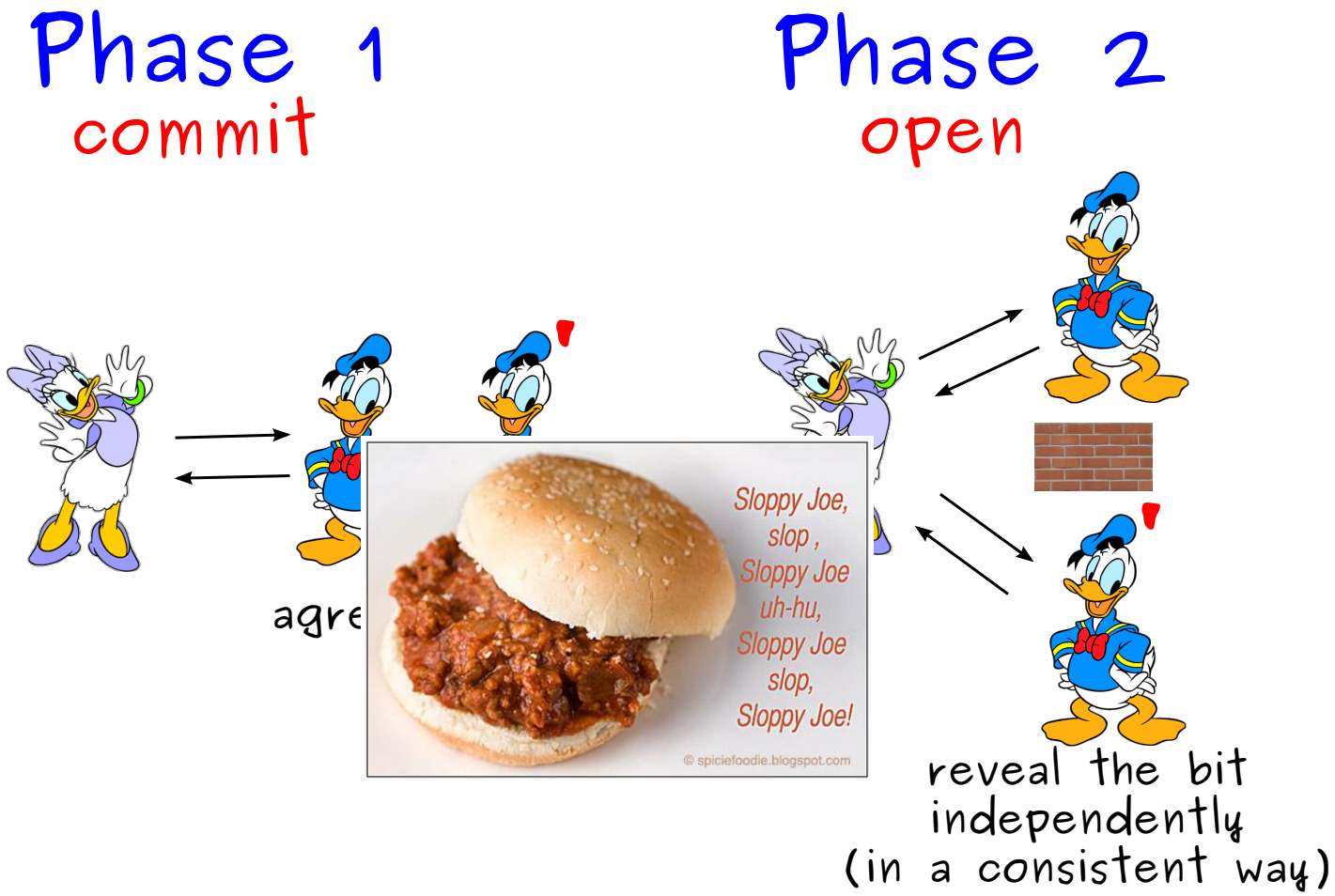
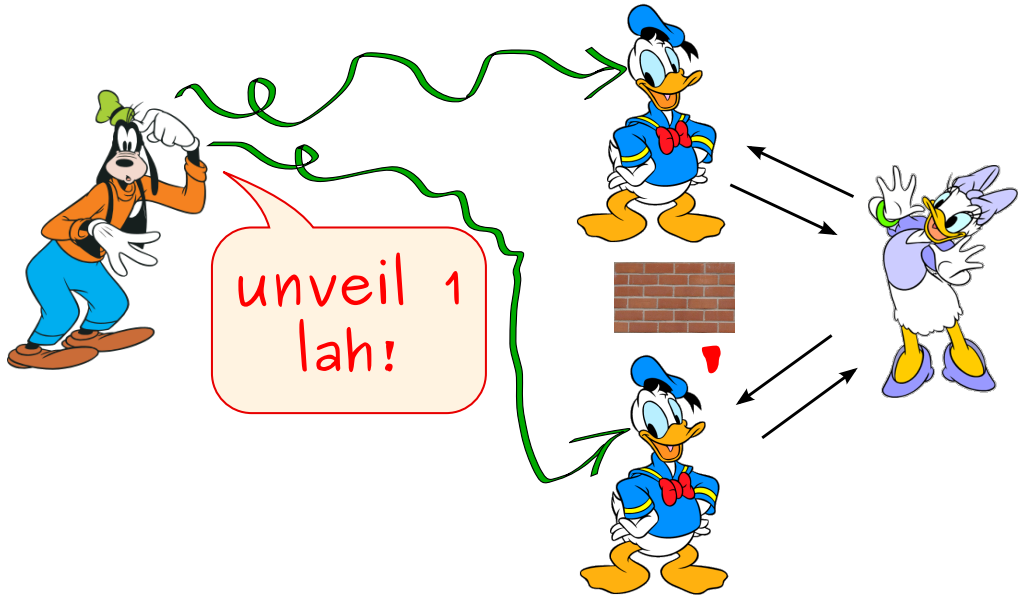


Local vs. global command

Local vs. global command



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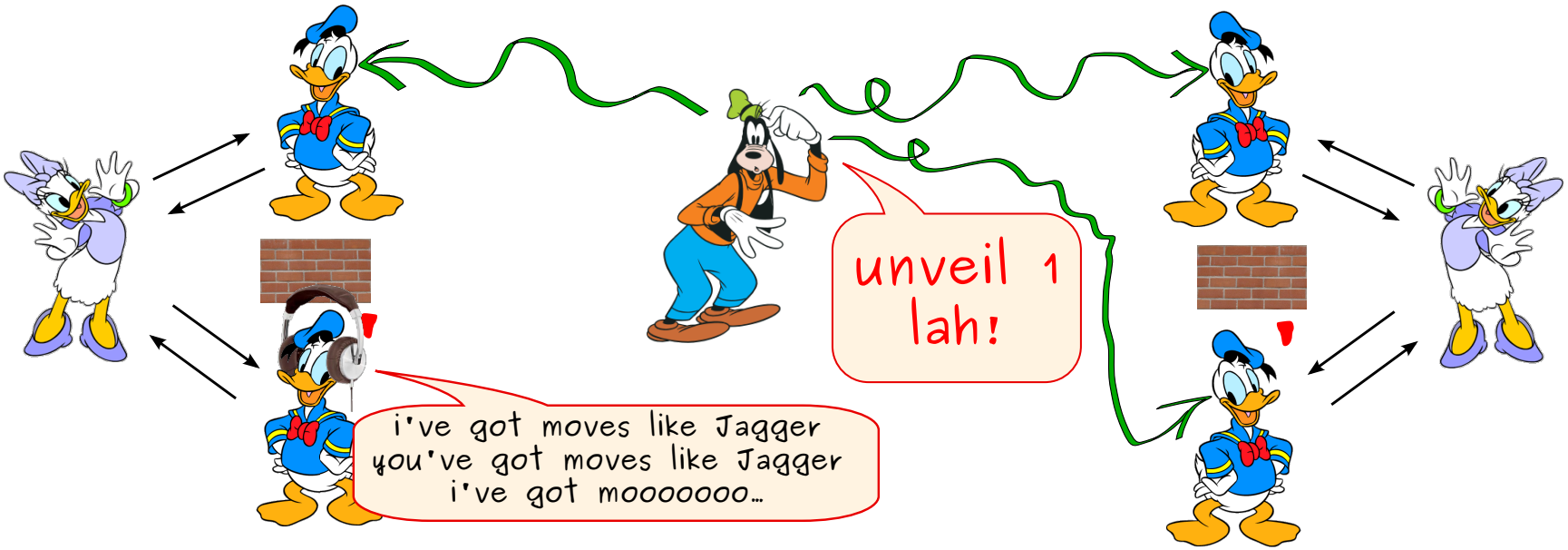

Local vs. global command

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local command

trivial protocol is secure
(directly from no-signalling)

global command

no classical protocol
can be secure

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what about quantum ???

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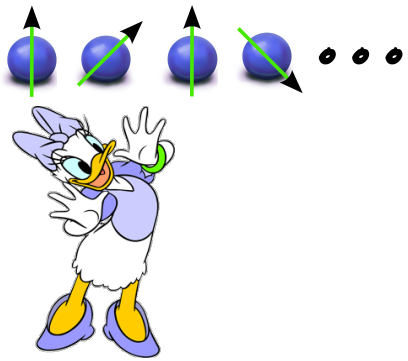
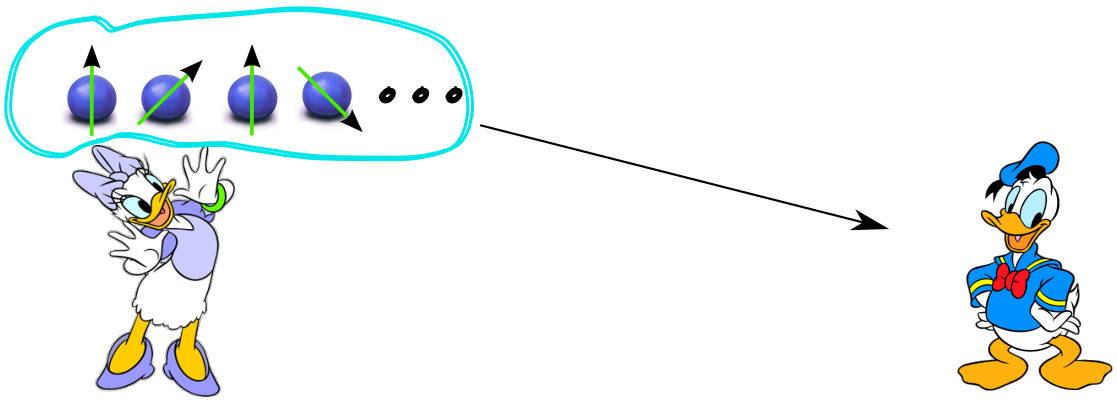
RBC [Kent'11] – Commit phase



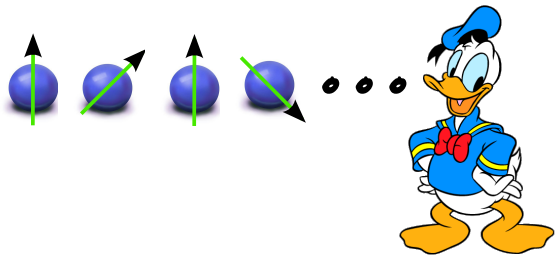
Alice creates n BB84 states ...

RBC [Kent'11] – Commit phase



. . . and sends them to Bob.

RBC [Kent'11] – Commit phase



Bob receives the qubits . . .

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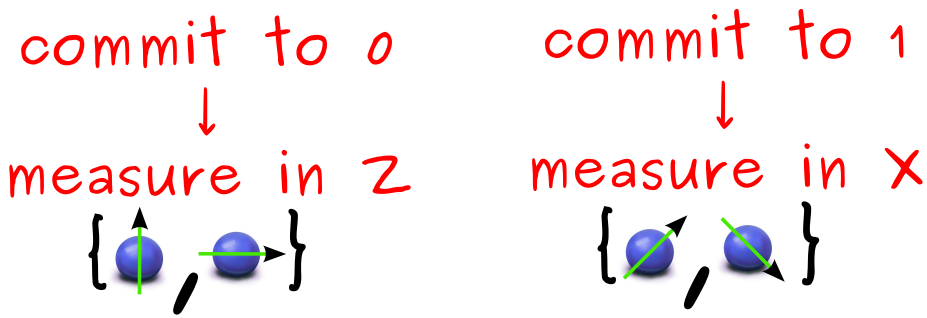
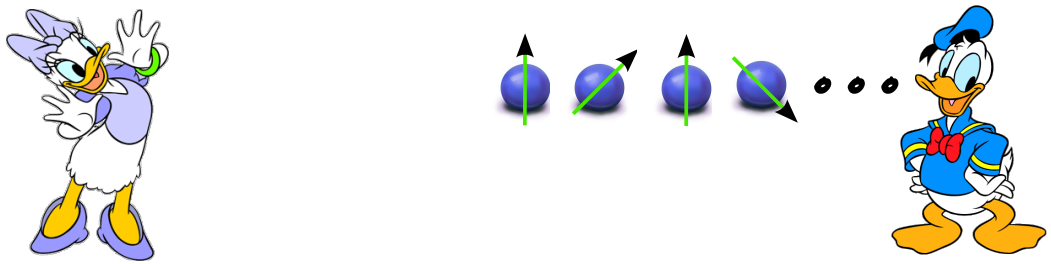




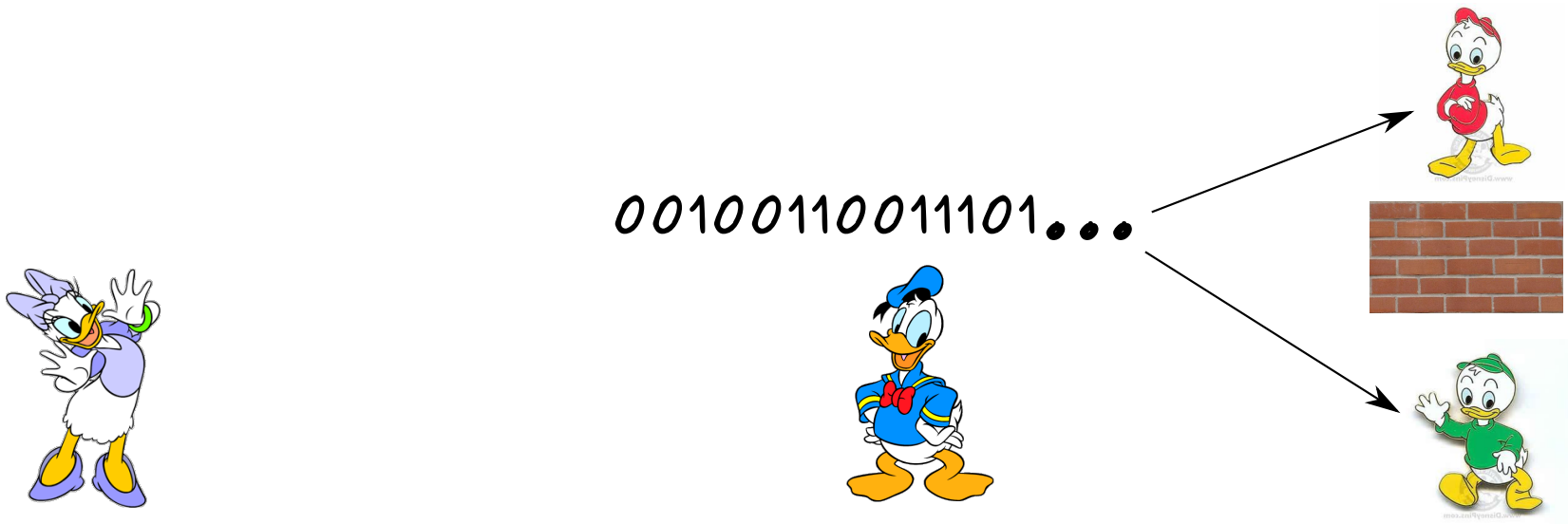
. . . and measures them in either computational or Hadamard basis.

RBC [Kent'11] – Commit phase

00100110011101...

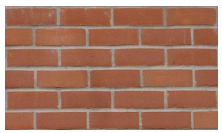
Bob obtains a bit string . . .

RBC [Kent'11] – Commit phase

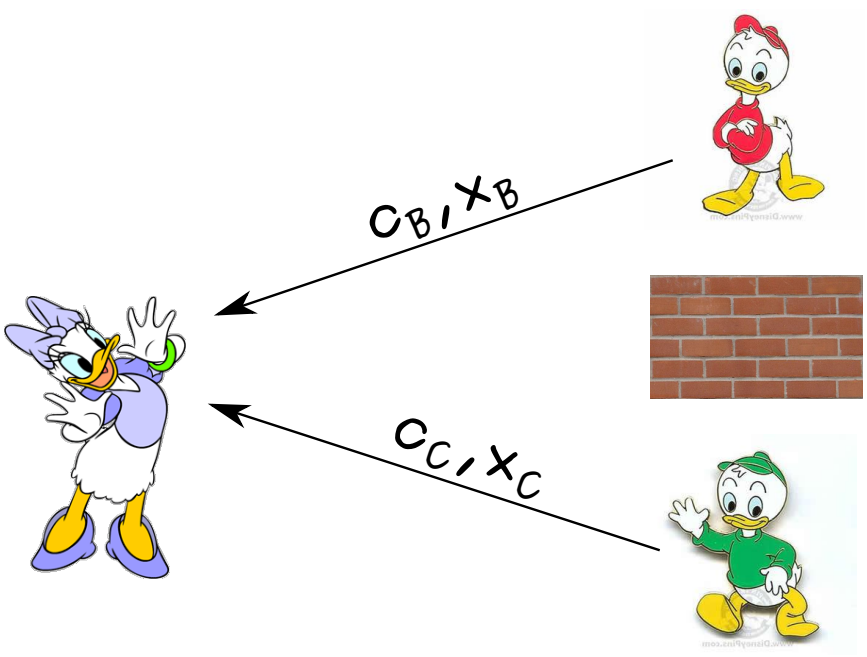


. . . and sends it to his agents.

RBC [Kent'11] – Open phase



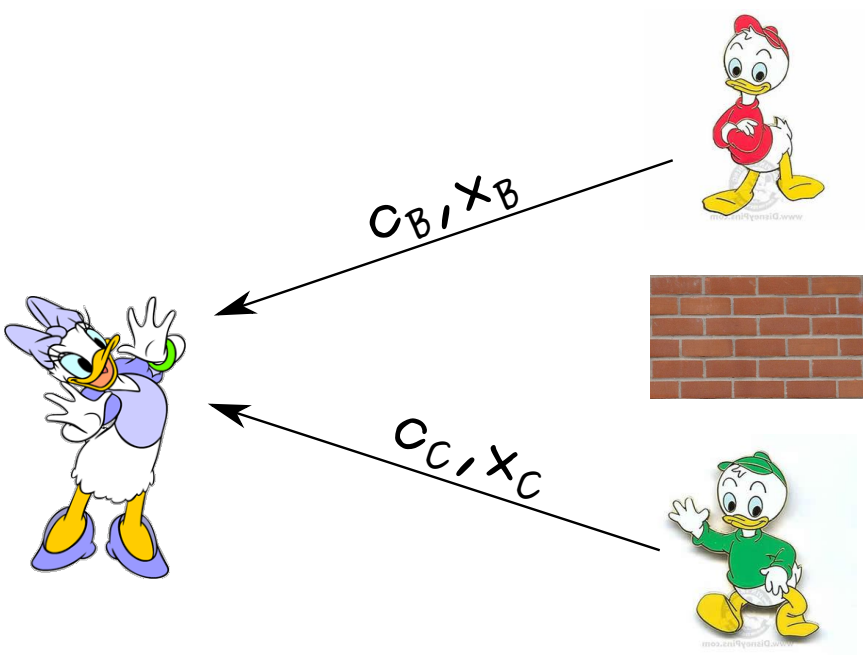
RBC [Kent'11] – Open phase



Alice checks if

- $c_B \stackrel{?}{=} c_C$,
- x_B is consistent with the BB84 states,
- x_C is consistent with the BB84 states.

RBC [Kent'11] – Open phase



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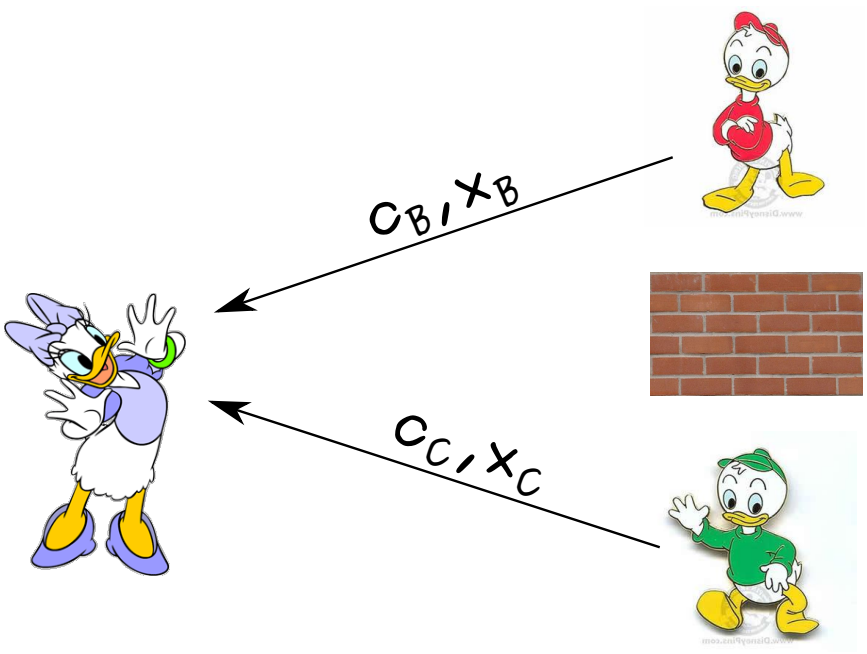

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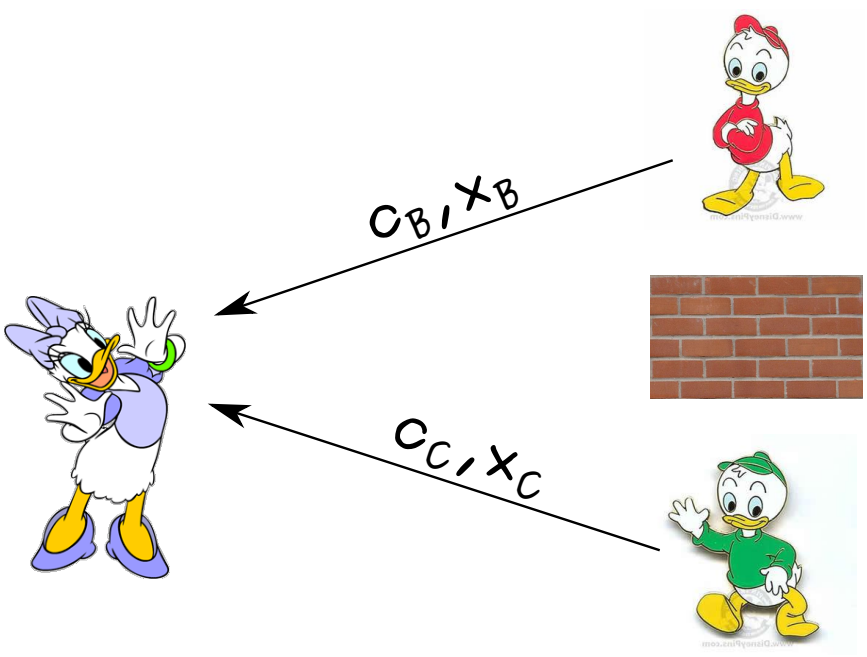
RBC [Kent'11] – Open phase



Alice checks if

- $c_B \stackrel{?}{=} c_C$,
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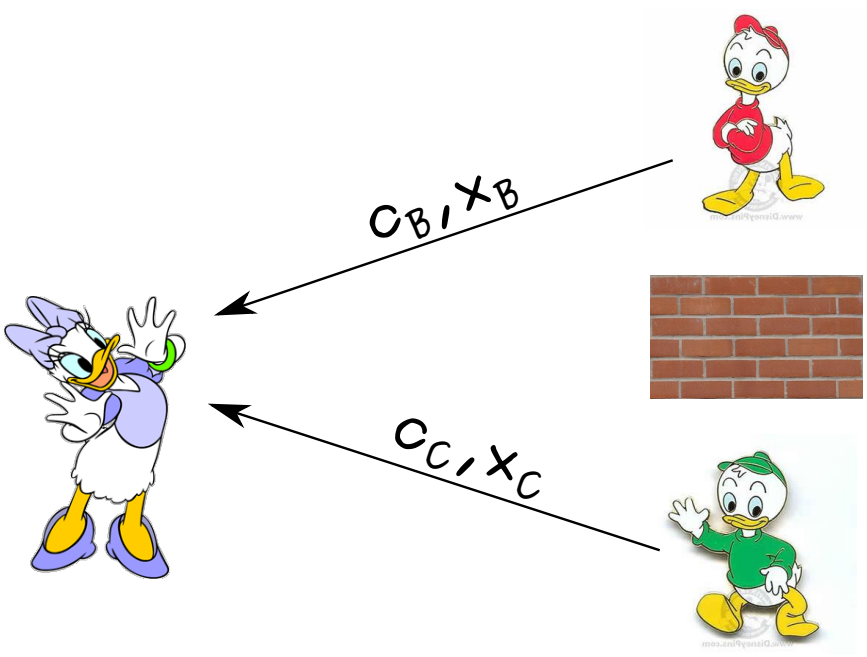

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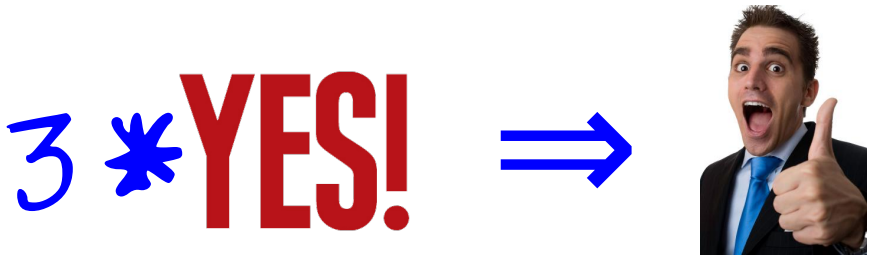
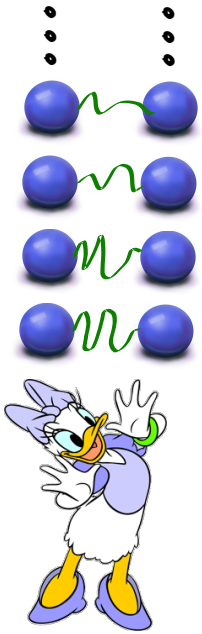
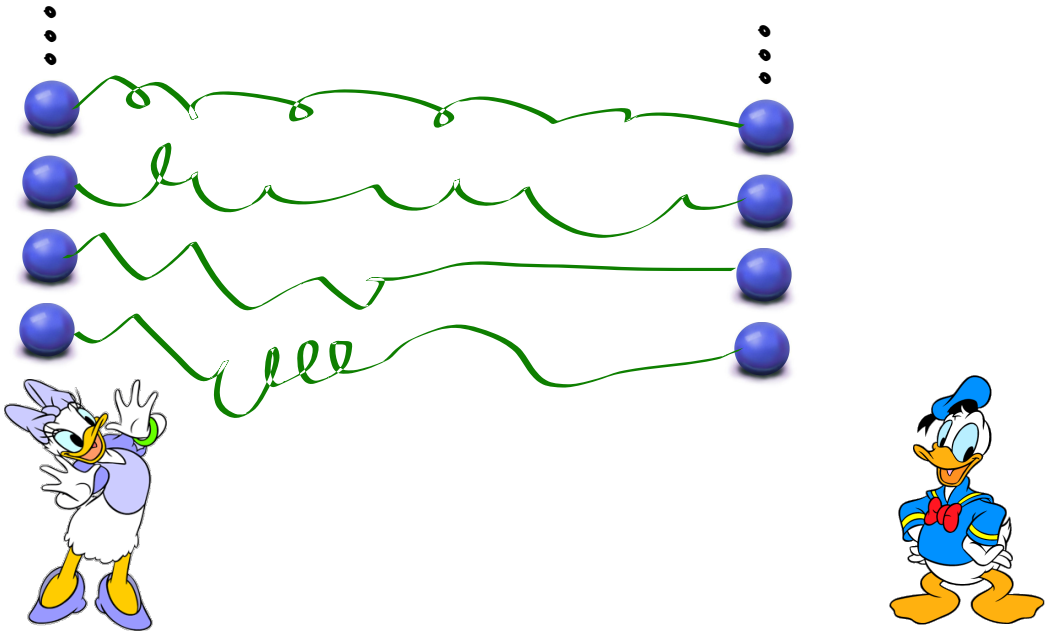


Purified RBC [Kent'11]



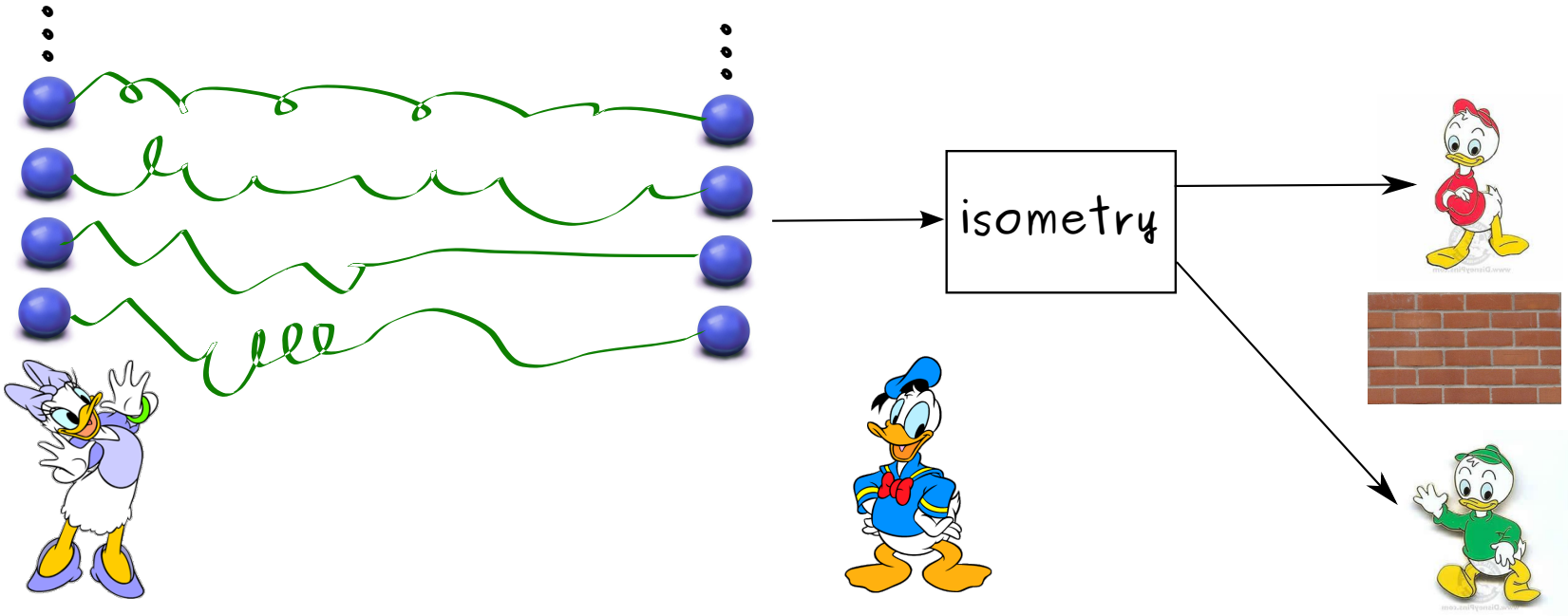
Alice creates n EPR pairs . . .

Purified RBC [Kent'11]



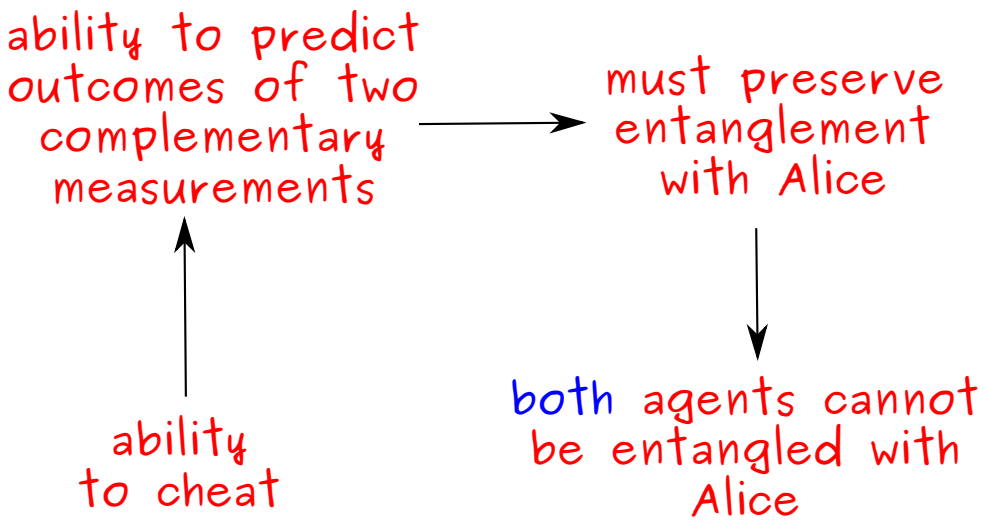
. . . and sends half of each to Bob.

Purified RBC [Kent'11]



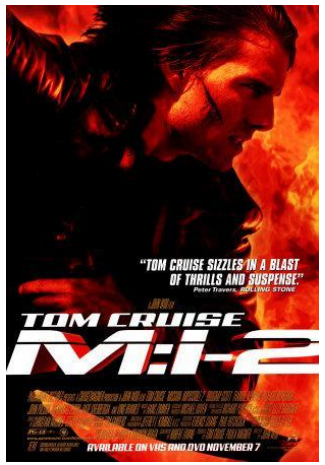
Bob applies an arbitrary isometry which splits the system into two parts. Each agent receives one of them.

RBC [Kent'11] – intuition why it is secure



RBC [Kent'11] – intuition why it is secure

ability to predict outcomes of two complementary measurements → must preserve entanglement with Alice → both agents cannot be entangled with Alice → ability to cheat



Outline

- Two-party cryptographic primitives
- Classical and quantum bit commitment
- Relativistic setting
- Relativistic bit commitment protocol [Kent'11]
- Security proof
- Summary and open questions

Security proof – no-signalling

			Charlie			
			$c = 0$		$c = 1$	
			accept	reject	reject	accept
Bob	$b = 0$	accept	p_0	a_{12}	$\cdot$	α
		reject	a_{21}	a_{22}	a_{23}	a_{24}
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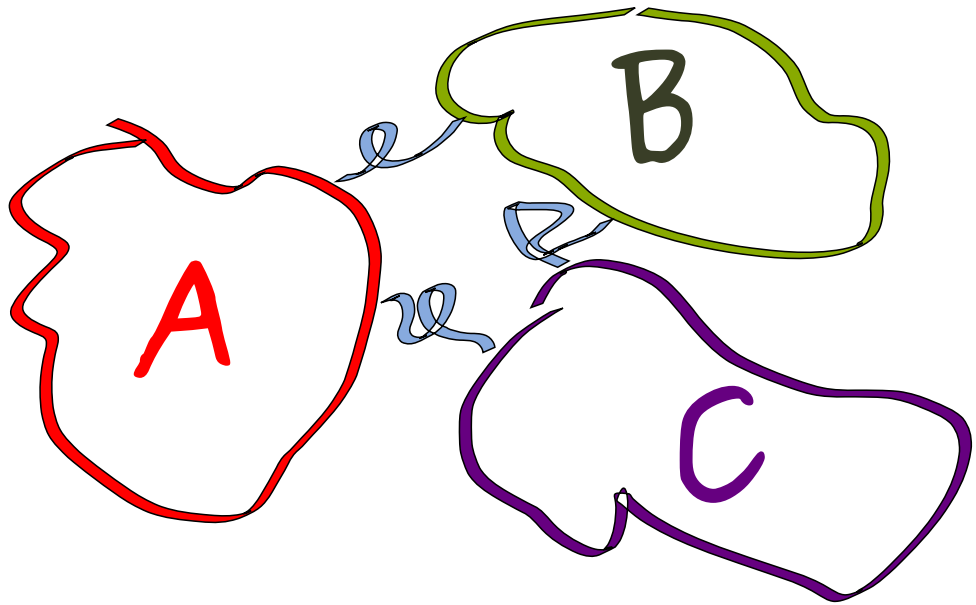
GOAL : get a bound on α

Security proof – uncertainty relation [TR'11]



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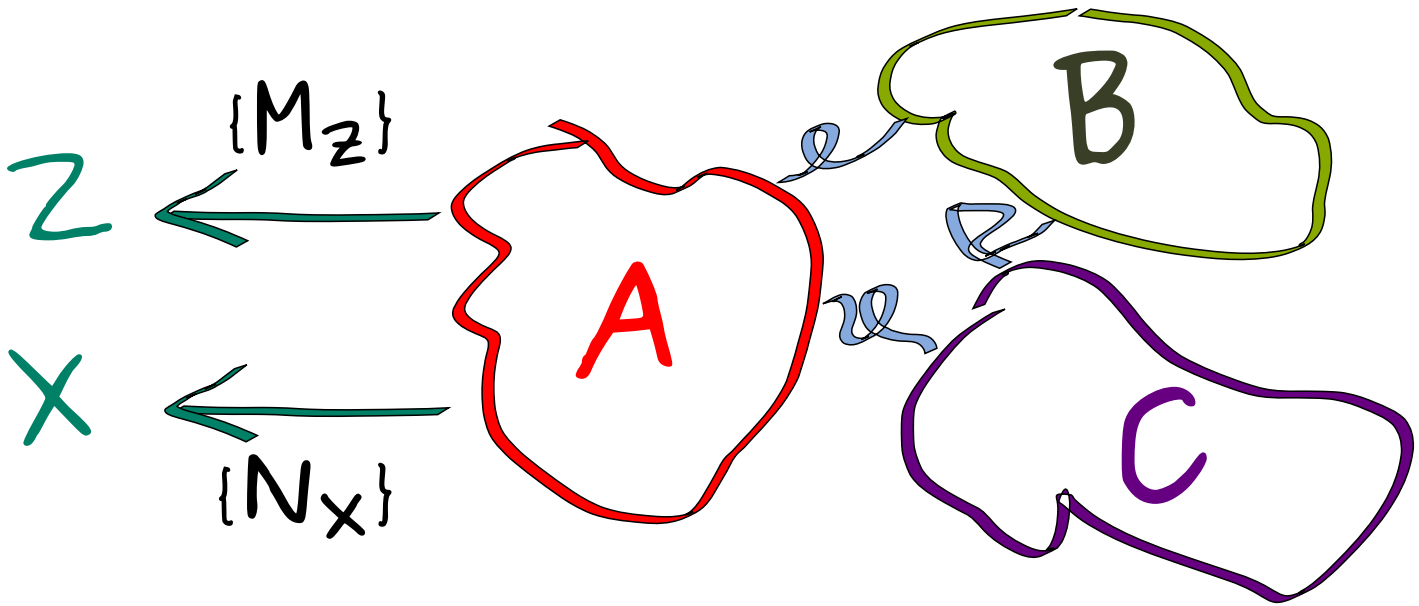

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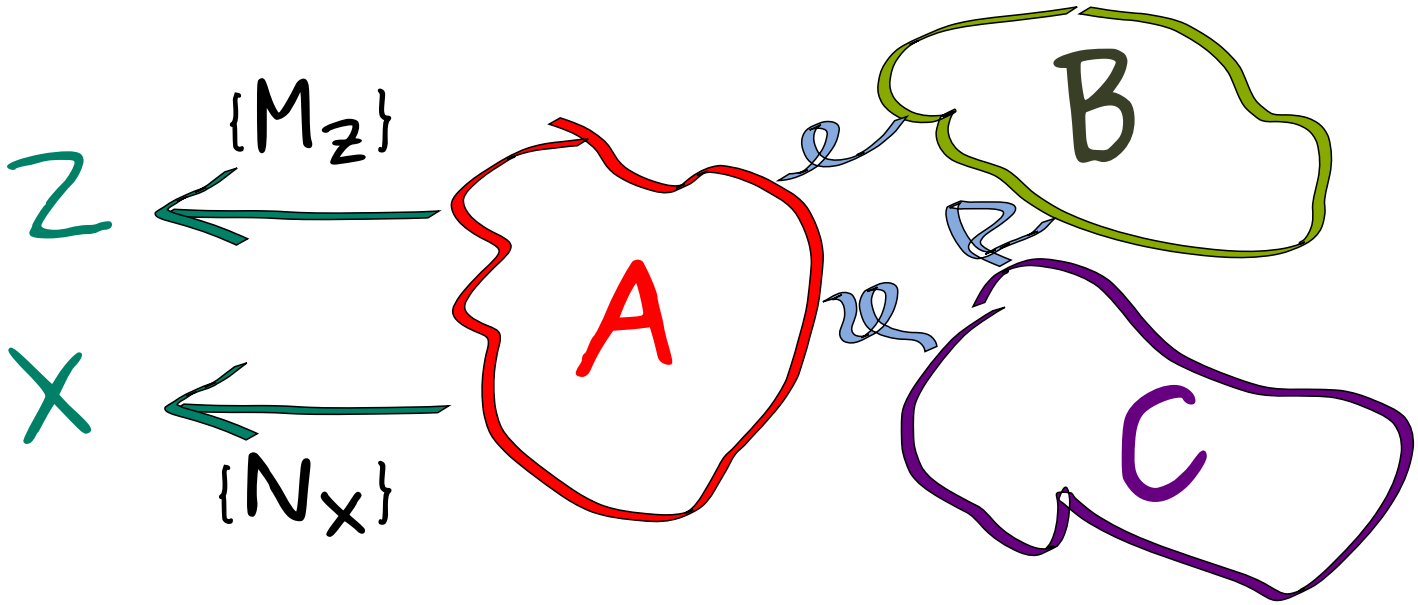

$$\mathrm{H}_{\max}(Z|B) + \mathrm{H}_{\min}(X|C) \geq \log \frac{1}{c},$$

where $c := \max_{z,x} \|\sqrt{M_z}\sqrt{N_x}\|_\infty^2$.

Security proof – sketchy sketch

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Bob is able to guess Z_r with high probability → his knowledge about Z_r must be significant

↓ (the sampling is random)

his knowledge about Z_g must also be significant → $H_{max}(Z_g|B)$ must be low

↓ (uncertainty relation)

$H_{min}(X_g|C)$ must be high → Charlie cannot guess X_g

Security proof – main result

Doing the maths properly gives

$$\alpha \leq 2^{1-n(1-h(\delta))} + 2\exp\left(-\frac{1}{2}n\delta^2\right),$$

for any $0 < \delta < \frac{1}{2} \implies$ **exponential decay**.

The fastest decay rate is achieved for $\delta \approx 0.33$, $\alpha \sim 2^{-0.08n}$

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- what about introducing some **noise tolerance**? (crucial if we think about doing an experiment)
- we know that RBC cannot be universally composable but maybe some weaker notion of **composability** holds. can we get **string commitment** by executing it multiple times (sequentially or in parallel)?

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