Advanced quantum information: entanglement and nonlocality

Lecture 8

- ask questions at any time using voice or chat (during the class and during the break)
- **please tell me if handwriting not legible**
- teaching material available at: http://...

$$P(ab|xy) = \sum_{\lambda}' q(\lambda)\, r_A(a|x,\lambda) \cdot r_B(b|y,\lambda)$$

Fine's thm

(1) $P(ab|xy) \in \mathcal{L}$

(2) Outcomes to all meas. are simultaneously defined

$P(ab|x=2, y=1) \equiv P(a_2 b_1)$

$\exists\, P(a_1 a_2 \cdots a_k\; b_1 \cdots b_k)$

$$\sum_{\substack{a_1, a_3 \ldots \\ b_2 \ldots}} P(a_1 a_2 \cdots b_1 \cdots b_k) = P(a_2 b_1)$$

(1) $\Leftrightarrow$ (2)

$\Rightarrow$

$P(a_1 \cdots a_k\, b_1 \cdots b_k) = \sum_{\lambda \in L}' q(\lambda) \cdot \prod_{j=1}^{k} r_A(a_j|j,\lambda) \cdot r_B(b_j|j,\lambda)$

(2) $\Rightarrow$ (1)

$\lambda = (a_1 \cdots a_k\, b_1 \cdots b_k)$

a_2

$\mathcal{H}_A \otimes \mathcal{H}_B$

$\varrho_{AB}: \; \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$

$\varrho = \varrho^{\dagger} \quad \varrho \geq 0 \quad \mathrm{tr}\,\varrho = 1$

$\{P_a^x\}_{a=1}^{n}$ $\quad \{1, \ldots, k\}$

$\sum_{a=1}^{n} P_a^x = \mathbb{1} \quad \forall x \in [k]$

$\{Q_b^y\}_{b=1}^{n}$

$\{\varrho_{AB}, \{P_a^x\}, \{Q_b^y\}\}$ quantum realisation

$P(ab|xy) = \mathrm{tr}\left[(P_a^x \otimes Q_b^y)\, \varrho_{AB}\right]$

$\mathcal{Q}_{fin}$:= "P's obtainable with finite-dim. realisations"

$\mathcal{Q} = \mathrm{clos}(\mathcal{Q}_{fin})$ ← quantum set of correlations

$\mathcal{Q}$ is defined for specific k, n but the dimension of $\mathcal{H}_A, \mathcal{H}_B$ is irrelevant

(1) $\mathcal{L} \subseteq \mathcal{Q}$

$P \in \mathcal{L} \Rightarrow P \in \mathcal{Q}$

$\Updownarrow\; \{q(\lambda), r_A, r_B\}$

$\lambda \in L \qquad \mathbb{C}^d \otimes \mathbb{C}^d \qquad d = |L|$

$\{|e_\lambda\rangle\}_{\lambda \in L}$

$$\varrho_{AB} = \sum_{\lambda \in L}' q(\lambda)\, |e_\lambda\rangle\langle e_\lambda|_A \otimes |e_\lambda\rangle\langle e_\lambda|_B$$

$$P_a^x = \sum_{\lambda \in L} r_A(a|x,\lambda)\, |e_\lambda\rangle\langle e_\lambda|$$

$$Q_b^y = \sum_{\lambda \in L}' r_B(b|y,\lambda)\, |e_\lambda\rangle\langle e_\lambda|$$

(2) every mixed state can be purified

$\varrho_{AB} \to |\psi\rangle_{ABB'}$

$Q_b^y \to Q_b^y \otimes \mathbb{1}_{B'}$

(3) non-proj. measurements can be dilated to proj. meas.

No-signalling

$P(ab|xy)$

$$\left[\sum_b P(ab|xy) = \sum_b P(ab|xy')\right] \qquad y, y' \in [k],\; a \in [n]$$

$P(a|x) = \sum_b P(ab|xy)$

marginal distribution

NS ← "P's which satisfy NS conditions"

$\mathcal{L}: \sum_b \sum_{\lambda \in L} q(\lambda)\, r_A(a|x,\lambda)\, r_B(b|y,\lambda)$ — $\sum_b r_B(b|y,\lambda) = 1$

$= \sum_\lambda q(\lambda)\, r_A(a|x,\lambda)$ independent of y

$\mathcal{Q}: \sum_b \mathrm{tr}\; P_a^x \otimes \sum_b Q_b^y\; \varrho_{AB}$ — $\sum_b Q_b^y = \mathbb{1}$

$= \mathrm{tr}\left[(P_a^x \otimes \mathbb{1})\varrho_{AB}\right] = \mathrm{tr}\left(P_a^x \varrho_A\right)$

$\mathcal{L} \subsetneq \mathcal{Q} \subsetneq NS$

Remark:

LR: all props. should be well-defined simultaneously

↕ inconsistency

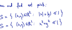
Q: every* meas. disturbs the system

Convex geometry

$S \subseteq \mathbb{R}^d$

(1) S is convex

$x, y \in S \Rightarrow px + (1-p)y \in S \quad \forall p \in [0,1]$



(2) $z \in S$ extend

$px + (1-p)y = z \Rightarrow x = y = z$

$p \in (0,1)$

Draw and find ext. points:

$S = \{(x,y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$

$S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$

$(x,y) \in \mathbb{R}^2 : \sqrt{|x|} + \sqrt{|y|} \leq 1$

Krein-Milman's thm

Every compact convex set is equal to the convex hull of its extremal points.

↑ closed and bounded

$S = \mathbb{R}, \quad S = (0,1) \subseteq \mathbb{R}$

$P(ab|xy) \quad P \in \mathbb{R}^{n^2 k^2}$

$\mathcal{L}, \mathcal{Q}, NS$ ⊂ subsets of $\mathbb{R}^{n^2 k^2}$

Goal: show that they are convex and compact

$P_0(ab|xy) = \sum_{\lambda \in L_0} q_0(\lambda)\, r_{A,0}(a|x,\lambda)\, r_{B,0}(b|y,\lambda)$

$P_1(ab|xy) = \sum_{\lambda \in L_1} q_1(\lambda)\, r_{A,1}(a|x,\lambda)\, r_{B,1}(b|y,\lambda)$

$L_0 \cap L_1 = \emptyset$

$L = L_0 \cup L_1 \qquad p \in [0,1]$

$$q(\lambda) = \begin{cases} p \cdot q_0(\lambda) & \lambda \in L_0 \\ (1-p)\, q_1(\lambda) & \lambda \in L_1 \end{cases}$$

$$r_A(a|x,\lambda) = \begin{cases} r_{A,0}(a|x,\lambda) & \lambda \in L_0 \\ r_{A,1}(a|x,\lambda) & \lambda \in L_1 \end{cases}$$

$r_B(b|y,\lambda)$

$\Rightarrow p \cdot P_0 + (1-p) P_1$

$\mathcal{L}$ is convex!