$\mathcal{H}$ – Hilbert space

$|\psi\rangle$ - pure state

$\rho, \sigma \in \mathcal{L}(\mathcal{H})$ mixed state

measurement on $\mathcal{H}$ with n outcomes

$\{F_j\}_{j=1}^n \quad F_j \in \mathcal{L}(\mathcal{H})$

$F_j \geq 0, \quad \sum_{j=1}^n F_j = \mathbb{1}$ $\quad$ $\boxed{F_j = K_j^\dagger K_j}$

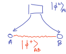
$\{K_j\}_{j=1}^n \quad \sum_{j=1}^n K_j^\dagger K_j = \mathbb{1}$

$$\rho \longmapsto \sum_{j=1}^n K_j \rho K_j^\dagger \otimes |j\rangle\langle j|$$

↑ postmeasurement state $\quad$ ↑ classical outcome

Historical introduction

EPR 1935



$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice measures in $\{|0\rangle, |1\rangle\}$
$\{|+\rangle, |-\rangle\}$

$\Rightarrow$ Bob's system conditioned on her outcome in $|0\rangle$ or $|1\rangle$
$|+\rangle$ or $|-\rangle$

$\Rightarrow$ Alice can perfectly predict result of Bob's $\{|0\rangle, |1\rangle\}$
$\{|+\rangle, |-\rangle\}$

$\Rightarrow$ there should exist objective outcomes
observer-independent

not consistent with uncertainty principle

EPR : QM formalism not complete

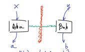
modern : QM does not satisfy local realism

realism : objects have properties (regardless of measurement)

local : these properties should be affected by events far away with no time-delay

Bell experiment

Bell 1964



$x, y \in [k] := \{1, 2, \ldots, k\}$
$a, b \in [n]$

- boxes cannot communicate
- can repeat many times, boxes are reliable

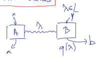
$P(ab|xy) \leftarrow$ well-defined, can be estimated to arbitrary precision

$P = \{P(ab|xy)\}_{abxy} \in \mathbb{R}^{n^2k^2}$

probability point

What happens if I fix the physical theory?

Classical devices



λ - shared random variable

$r_A(a|x, \lambda) : [n] \times [k] \times L \longrightarrow \mathbb{R}_+$

$\sum_a r_A(a|x,\lambda) = 1$

$r_B(b|y, \lambda)$

$$\boxed{P(ab|xy) = \sum_{\lambda \in L} q(\lambda) \cdot r_A(a|x,\lambda) \cdot r_B(b|y,\lambda)}$$

"P is local" $\quad$ mathematical definition of local realism

"P admits a local hidden variable (LHV) model"

$\mathcal{L}$ – set of all P's of the form above

(1) $|L| = 1$

$P(ab|xy) = r_A(a|x) \cdot r_B(b|y)$

uncorrelated

$\mathcal{L}$ = "convex combinations of uncorrelated distributions"
(cf separable states)